Homework # 3

Chapter 38
Photons, Electrons & Atoms
38.4 A laser used to weld detached retinas emits light with a wavelength of 650 nm in pulses that are 25.0 ms in duration. The average power during each pulse is 0.6 W.

(a) How much energy is in each pulse in joules?
(b) How much energy is in each pulse in electron volts?
(c) What is the energy of one photon in joules?
(d) What is the energy of one photon in electron volts?
(e) How many photons are in each pulse?

\[
\begin{align*}
\text{(a)} & \quad E = P \Delta t = (0.6 \text{ W})(25 \text{ ms}) = 15 \text{ mJ} \\
\text{(b)} & \quad 15 \times 10^{-3} \text{ J} \left(1 \text{ eV}/1.6 \times 10^{-19} \text{ J}\right) = 9.4 \times 10^{16} \text{ eV} \\
\text{(c)} & \quad E = hf = hc/\lambda = (2 \times 10^{-25} \text{ J-m})/(650 \times 10^{-9} \text{ m}) = 3.06 \times 10^{-19} \text{ J} \\
\text{(d)} & \quad E = hf = hc/\lambda = (1.24 \text{ eV-\mu m})/(0.650 \text{ \mu m}) = 1.91 \text{ eV} \\
\text{(e)} & \quad \text{# of photons} = \text{total energy/energy per photon} \\
& \quad N = 15 \text{ mJ} / (3.06 \times 10^{-19} \text{ J}) = 4.9 \times 10^{16} \text{ photons} \\
\end{align*}
\]

\[
\begin{align*}
\text{hc} & = (6.63 \times 10^{-34} \text{ J-s})(3 \times 10^{8} \text{ m/s}) = 2 \times 10^{-25} \text{ J-m} = 1.24 \text{ eV-\mu m}
\end{align*}
\]
A clean nickel surface is exposed to light with a wavelength of 209 nm. The photoelectric work functions for nickel is 5.10 eV. What is the maximum speed of the photoelectrons emitted from this surface?

Use $3 \times 10^8$ m/s for the speed of light in a vacuum, $6.626 \times 10^{-34}$ J-s for Planck's constant, and $9.11 \times 10^{-31}$ Kg for the mass of an electron.

$$KE = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{(1.24 \text{ eV} \cdot \mu \text{m})}{(0.209 \mu \text{m})} - 5.10 \text{ eV} = 0.83 \text{ eV}$$

$$0.83 \text{ eV} \cdot (1.6 \times 10^{-19} \text{ J/eV}) = 13.3 \times 10^{-20} \text{ J}$$

$$KE = \frac{1}{2} mv^2$$

$$13.3 \times 10^{-20} = \frac{1}{2} (9.11 \times 10^{-31})v^2$$

$$v = 5.4 \times 10^5 \text{ m/s}$$
The photoelectric work function of potassium is 2.3 eV. Light having a wavelength of 260 nm falls on potassium.

(a) Find the stopping potential for light of this wavelength.
(b) Find the kinetic energy of the most energetic electrons ejected.
(c) Find the speeds of these electrons.

(a) \[ KE = hf - \phi = \frac{hc}{\lambda} - \phi = \frac{(1.24 \text{ eV-\mu m})}{(0.260 \text{ \mu m})} - 2.3 \text{ eV} = 2.47 \text{ eV} \]
    Stopping potential \( E_o = KE_{\text{max}} = 2.25 \text{ eV} \)

(b) \( KE = 2.47 \text{ eV} \)

(c) \( 2.47 \text{ eV} \times (1.6 \times 10^{-19} \text{ J/eV}) = 3.95 \times 10^{-19} \text{ J} \)

\[ KE = \frac{1}{2}mv^2 \]
\[ 3.95 \times 10^{-19} = \frac{1}{2} (9.11 \times 10^{-31})v^2 \]
\[ v = 9.3 \times 10^5 \text{ m/s} \]
38.20 An 4.87-MeV alpha particle from a radium $^{226}$Ra decay makes a head-on collision with a uranium nucleus. A uranium nucleus has 92 protons.

(a) What is the distance of closest approach of the alpha particle to the center of the nucleus? Assume that the uranium nucleus remains at rest and that the distance of closest approach is much larger than the radius of the uranium nucleus.

(b) What is the force on the alpha particle at the instant when it is at the distance of closest approach?

(a) $U(r) = k \frac{q_1 q_2}{r} = KE$

$(9 \times 10^9)(2)(92)(1.6 \times 10^{-19})^2 / r = (4.87 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})$

$r = 5.44 \times 10^{-14} \text{ m}$

(b) $F = -k \frac{q_1 q_2}{r^2} = -U/r$

$|F| = (4.87 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})/(5.44 \times 10^{-14} \text{ m})$

$|F| = 14.3 \text{ N}$
38.31 A large number of neon atoms are in thermal equilibrium.

(a) What is the ratio of the number of atoms in a 5s state to the number in a 3p state at 260 K?

(b) What is the ratio of the number of atoms in a 5s state to the number in a 3p state at 520 K?

(c) What is the ratio of the number of atoms in a 5s state to the number in a 3p state at 1000 K? The energies of these states are shown in Fig. 38.24a in the textbook.

(d) At any of these temperatures, the rate at which a neon gas will spontaneously emit 632.8-mm radiation is quite low. Explain why.

\[
\frac{n_{5s}}{n_{3p}} = e^{-(E_{5s} - E_{3p})/kT}
\]

From Fig. 38.24a in the textbook
\[
E_{5s} = 20.66 \text{ eV and } E_{3p} = 18.70 \text{ eV}
\]

\[
E_{5s} - E_{3p} = 20.66 \text{ eV} - 18.70 \text{ eV} = 1.96 \text{ eV}(1.602 \times 10^{-19} \text{ J/1 eV}) = 3.140 \times 10^{-19} \text{ J}
\]

(a) \[
\frac{n_{5s}}{n_{3p}} = e^{-(3.4 \times 10^{-19} \text{ J})/(1.38 \times 10^{-23} \text{ J/K})(260 \text{K})} = e^{-94.8} = 7.0 \times 10^{-42}
\]

(b) \[
\frac{n_{5s}}{n_{3p}} = e^{-(3.4 \times 10^{-19} \text{ J})/(1.38 \times 10^{-23} \text{ J/K})(520 \text{K})} = e^{-47.4} = 2.6 \times 10^{-21}
\]

(c) \[
\frac{n_{5s}}{n_{3p}} = e^{-(3.4 \times 10^{-19} \text{ J})/(1.38 \times 10^{-23} \text{ J/K})(1000 \text{K})} = e^{-24.6} = 2.0 \times 10^{-11}
\]

(d) At each of these temperatures the number of atoms in the 5s excited state, the initial state for the transition that emits 632.8 nm radiation, is quite small. The ratio increases as the temperature increases.
X-rays with initial wavelength $6.65 \times 10^{-2}$ nm undergo Compton scattering.

(a) What is the largest wavelength found in the scattered X-rays?
(b) At which scattering angle is this wavelength observed?

\[
\lambda' - \lambda = \frac{h}{mc} (1 - \cos \phi) = \lambda_C (1 - \cos \phi)
\]

Solve for $\lambda'$: $\lambda' = \lambda + \lambda_C (1 - \cos \phi)$

(b) The largest $\lambda'$ corresponds to $\phi = 180^\circ$.

(a) $\lambda' = \lambda + 2\lambda_C = 0.0665 \times 10^{-9}$ m + $2(2.426 \times 10^{-12}$ m) = $7.135 \times 10^{-11}$ m = 0.0714 nm.
A photon scatters in the backward direction \((\theta = 180^\circ)\) from a free proton that is initially at rest. What must the wavelength of the incident photon be if it is to undergo a 10.0\% change in wavelength as a result of the scattering?

\[
\Delta \lambda = \frac{h}{mc\lambda} (1 - \cos \phi) \Rightarrow \lambda = \frac{h}{mc\lambda} \left( 1 - \cos \phi \right).
\]

\[
\lambda = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})}{(1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^{8} \text{ m/s})(0.100)} (1 + 1) = 2.65 \times 10^{-14} \text{ m}.
\]
38.54 (a) If the average frequency emitted by a 200 W light bulb is $5 \times 10^{14}$ Hz, and 10.0% of the input power is emitted as visible light, approximately how many visible-light photons are emitted per second? (b) At what distance would this correspond to $10^{11}$ visible-light photons per square centimeter per second if the light is emitted uniformly in all directions?

(a) \[
\frac{dN}{dt} = \frac{(dE/dt)}{(dE/dN)} = \frac{P}{hf} = \frac{(200 \text{ W})(0.10)}{h(5.00 \times 10^{14} \text{ Hz})} = 6.03 \times 10^{19} \text{ photons/sec}.
\]

Therefore, \[
\frac{(dN/dt)}{4\pi r^2} = 1.00 \times 10^{11} \text{ photons/sec} \cdot \text{cm}^2.
\]

(b) \[
r = \left(\frac{6.03 \times 10^{19} \text{ photons/sec}}{4\pi(1.00 \times 10^{11} \text{ photons/sec} \cdot \text{cm}^2)}\right)^{1/2} = 6930 \text{ cm} = 69.3 \text{ m}.
\]
\textbf{38.61} An incident x-ray photon with a wavelength of $9.30 \times 10^{-2}$ nm is scattered in the backward direction from a free electron that is initially at rest.

(a) What is the magnitude of the momentum of the scattered photon?

(b) What is the kinetic energy of the electron after the photon is scattered?

(a) The wavelength of the scattered photon is given by the Compton shift:

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos \phi)$$

with $\phi = 180^\circ$ so $\lambda' = \lambda + 2h/mc = \lambda + 2hc/mc^2 = 0.093 + 0.00485 = 0.098$ nm

The momentum of the scattered photon is $p' = h/\lambda' = 6.77 \times 10^{-24}$ kg-m/s

(b) Apply conservation of energy to the collision to calculate the kinetic energy of the electron after the scattering. The energy of the photon is given by Eq.(38.2),

$$E = E' + E_e; \frac{hc}{\lambda} = \frac{hc}{\lambda'} + E_e$$

$$E_e = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = hc(\frac{1}{\lambda} - \frac{1}{\lambda'})$$

$$= (1.24 \text{ eV-\mu m})[1/(9.3 \times 10^{-11}) - 1/(9.79 \times 10^{-11})] = 670 \text{ eV}$$

The energy of the incident photon is 13.8 keV, so only about 5% of its energy is transferred to the electron. This corresponds to a fractional shift in the photon’s wavelength that is also 5%.
A sample of hydrogen atoms is irradiated with light with a wavelength of 85.4 nm, and electrons are observed leaving the gas.

(a) If each hydrogen atom were initially in its ground level, what would be the maximum kinetic energy in electron volts of these photoelectrons?

(b) A few electrons are detected with energies as much as 10.2 eV greater than the maximum kinetic energy calculated in part (a). How can this be?

\[ hf = E_f - E_i, \]

\[ \frac{hc}{\lambda} = \frac{(6.626 \times 10^{-34} \text{ J} \cdot \text{s})(2.998 \times 10^8 \text{ m/s})}{85.5 \times 10^{-9} \text{ m}} \]

\[ \frac{hc}{\lambda} = 2.323 \times 10^{-18} \text{ J}(1 \text{ eV}/1.602 \times 10^{-19} \text{ J}) = 14.50 \text{ eV}. \]

\[ E_f = E_i + hf. \quad \text{ground state} \quad E_i = -13.60 \text{ eV}. \]

\[ E_f = -13.60 \text{ eV} + 14.50 \text{ eV} = 0.90 \text{ eV}. \]

(b) \[ n = 2 \quad -13.6 \text{ eV}/4 = -3.40 \text{ eV}, \quad 10.2 \text{ eV} \quad \text{greater than the energy of the ground state.} \]

If an electron with \( E = -3.40 \text{ eV} \) gains 14.5 eV from the absorbed photon, it will end up with

\[ 14.5 \text{ eV} - 3.4 \text{ eV} = 11.1 \text{ eV} \quad \text{of kinetic energy which is 10.2 eV greater than 0.90 eV.} \]