Moment Distribution

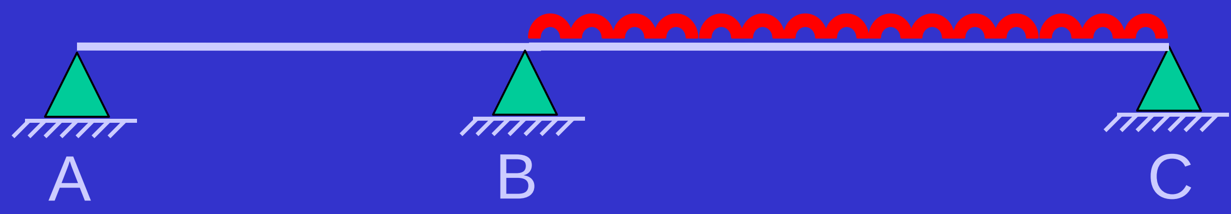
This is an alternative method to the slope-deflection method for analysing statically indeterminate plane-frame structures

The method is suitable for frame elements, where the principal deflection effects are caused by bending rather than shear or axial loading

The method makes use of the Principle of Superposition, fixed end moments and the slope-deflection equations

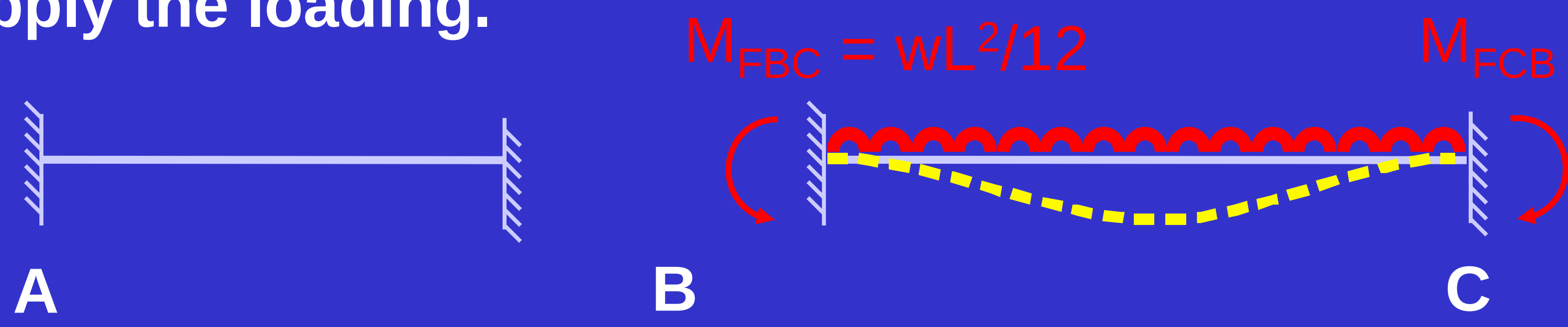
It is an iterative process that converges on an equilibrium solution…easier for hand calculations than the slope-deflection method

Moment Distribution



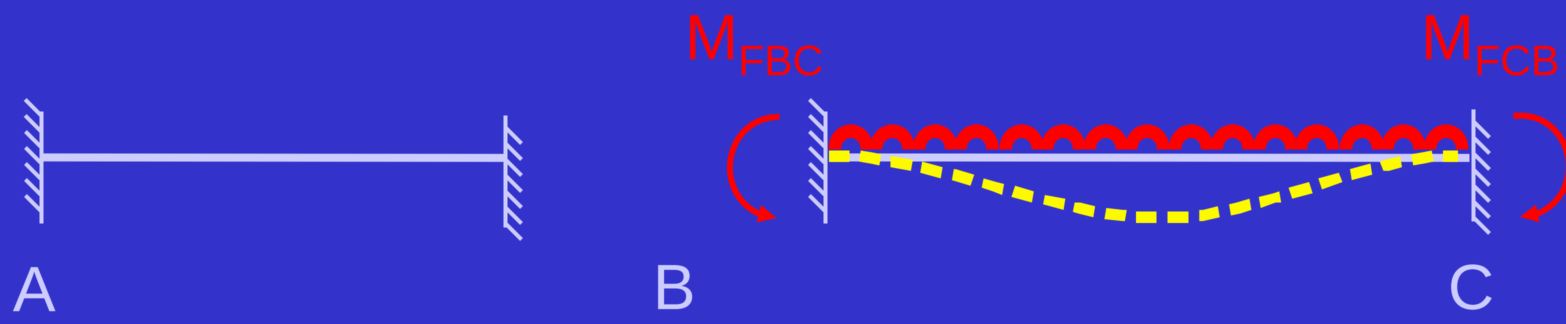
Consider a simple 2-span beam

First fully fix the ends of members AB and BC and apply the loading.



Hence find the fixed-end moments

Moment Distribution

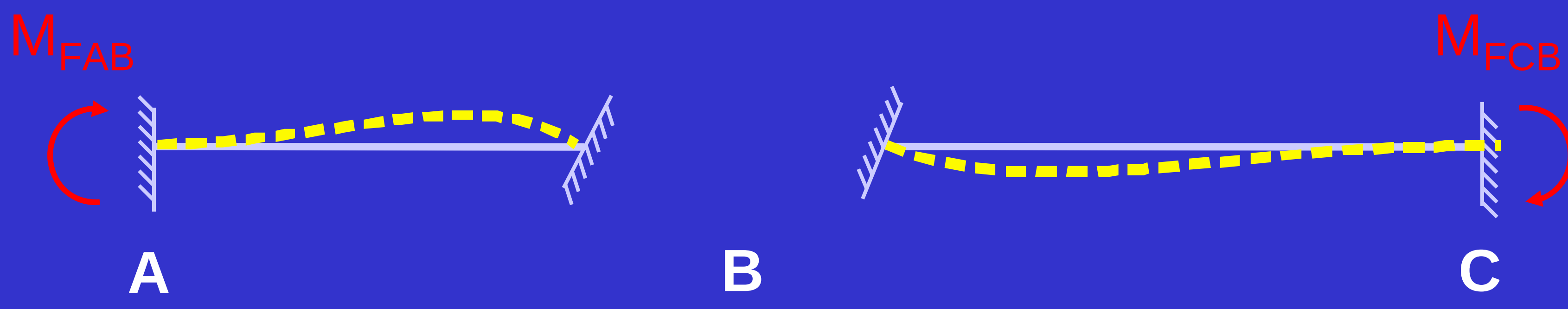


Joint B in this example wants to rotate in a clockwise direction but cannot.

Joint B is not in equilibrium. $M_{BA} = 0$ but $M_{BC} = M_{FBC}$.

The joint is out of balance

Moment Distribution



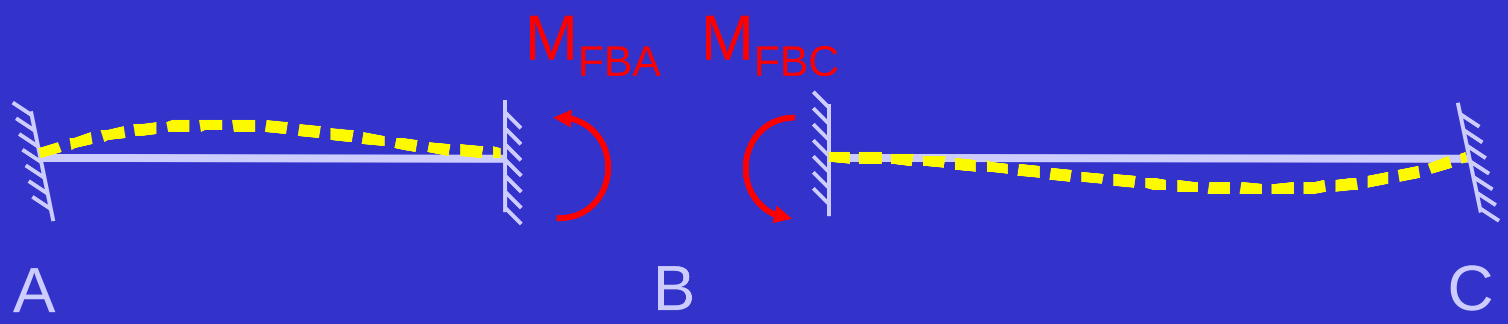
If we now release the fixity at Joint B and allow it to rotate then equilibrium is restored at this joint. The joint is balanced.

However this rotation now causes additional moments to the transferred to joints A and C, which are still fixed.

These moments are known as carry-over moments

The beam is still not in equilibrium… joint A and joint C would both like to rotate anti-clockwise, but cannot because they are fixed.

Moment Distribution



If we now fix joint B, and allow rotations at joints A & C (not the same rotation)

Then new carry-over moments are transferred back to joint B

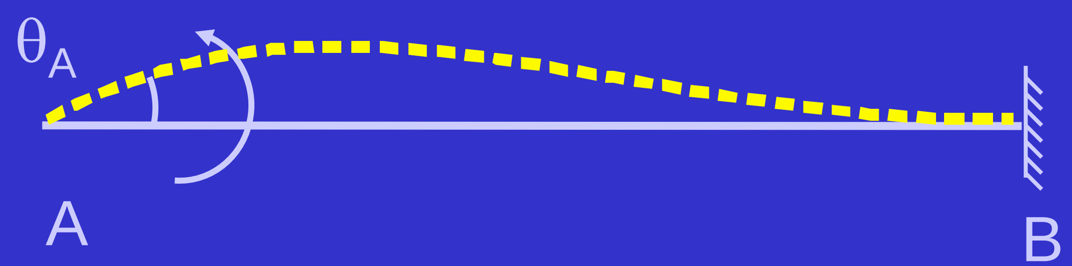
If the moments at joint B do not balance each other, then we must again fix joints A & C and allow joint B to rotate.

We continue with this and the carry-over moments will get smaller and smaller with each rotation until equilibrium is restored throughout

Moment Distribution

Stiffness, carry-over moments and distribution factors

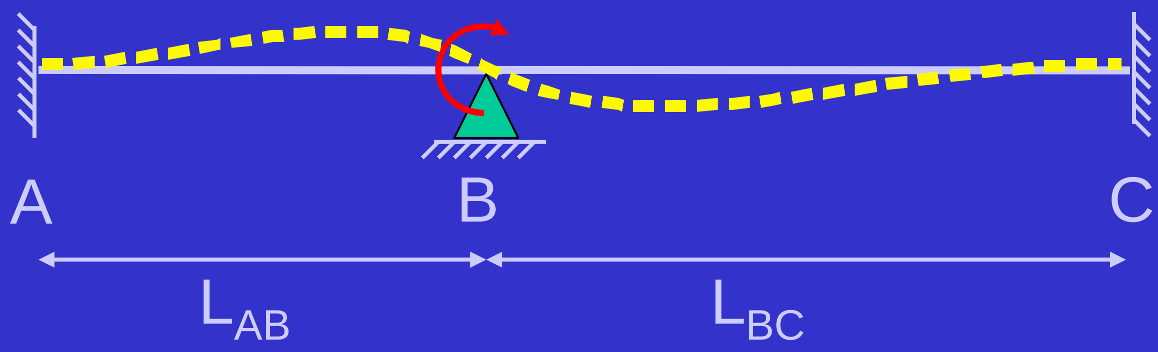
When a joint is released and allowed to rotate, the out of balance moment must be shared between the members on each side of the joint according to the bending stiffness of each member. If we force a positive (anticlockwise) rotation θ_A.



$$M_{DAB} = \frac{4EI\theta_A}{L_{AB}}$$

$$M_{DBA} = \frac{2EI\theta_A}{L_{AB}}$$

Moment Distribution



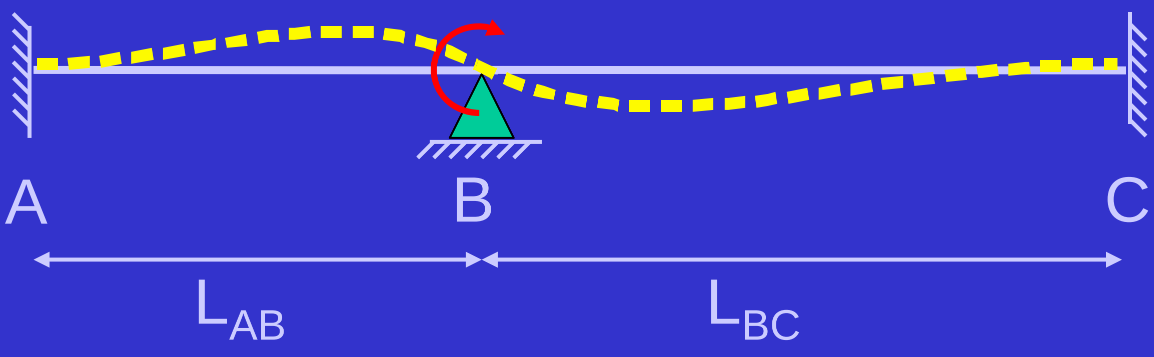
When joint B rotates by an angle θ_B, there are moments induced in AB and BC at B according to the bending stiffnesses of AB & BC.

These are known as the balancing moments

$$M_{DBA} = \frac{4EI\theta_B}{L_{AB}}$$

$$M_{DBC} = \frac{4EI\theta_B}{L_{BC}}$$

Moment Distribution



Thus equilibrium is restored to an out of balance joint by rotating the joint and adding moments to either side of the joint according to the rotational stiffness of the member on each side.

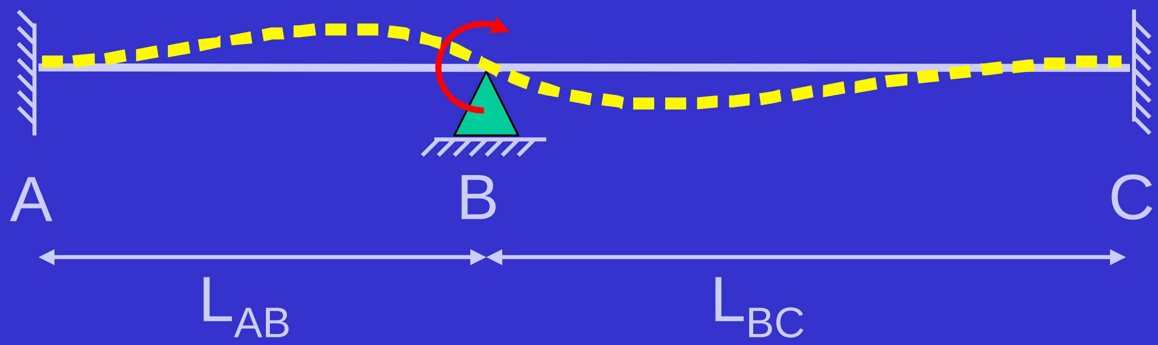
$$M_{DBA} = M_{O-O-B} \times \frac{1/L_{AB}}{1/L_{AB} + 1/L_{BC}}$$

$$M_{DBC} = M_{O-O-B} \times \frac{1/L_{BC}}{1/L_{AB} + 1/L_{BC}}$$

For constant EI

Distribution factor

Moment Distribution



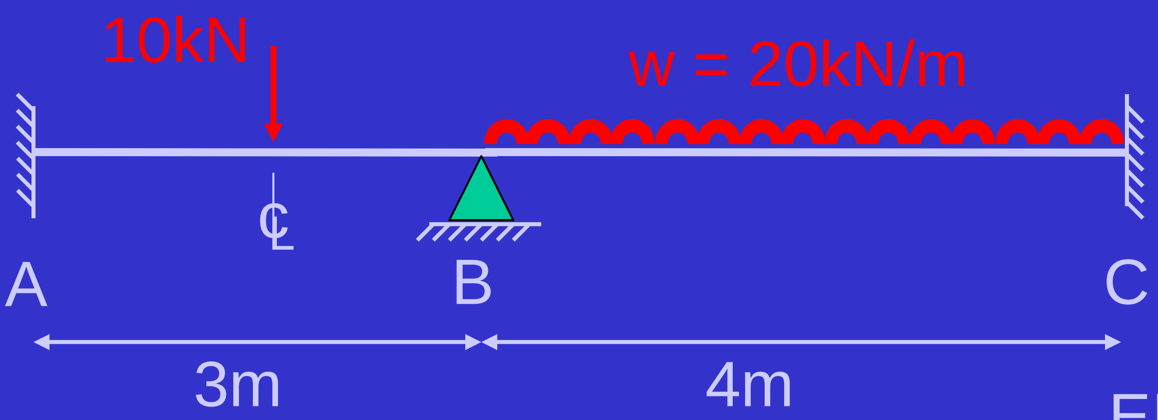
Half of the balancing moment is carried over to the joint at the other end of the member

$$M_{DAB} = \frac{M_{DBA}}{2}$$

$$M_{DCB} = \frac{M_{DBC}}{2}$$

Moment Distribution

So … now we are ready to try a numerical example:



EI = constant

Fixed end moments:

M_{FAB} = PL/8 = 3.75 kNm $\quad$ M_{FBA} = - 3.75 kNm

M_{FBC} = wL^2/12 = 26.7 kNm $\quad$ M_{FCB} = - 26.7 kNm

Distribution factors:

$$DF_{BA} = \frac{1/3}{1/3 + 1/4} = 0.57 \qquad DF_{BC} = \frac{1/4}{1/3 + 1/4} = 0.43$$

Moment Distribution

Carry out calculations using a moment distribution table

	A		B			C
DF		EI/L	0.57	0.43	EI/L	
	M_{AB}		M_{BA}	M_{BC}		M_{CB}
FEM	3.75		-3.75	26.7		-26.7

Moment Distribution

Carry out calculations using a moment distribution table

	A		B			C
DF		EI/L	0.57	0.43	EI/L	
	M_{AB}		M_{BA}	M_{BC}		M_{CB}
FEM	3.75		-3.75	26.7		-26.7

Out of balance
26.7-3.75=22.95

Moment Distribution

	A		B		C
DF			0.57	0.43	
	M_{AB}		M_{BA}	M_{BC}	M_{CB}
FEM	3.75		-3.75	26.7	-26.7
Bal			-13.1	-9.9	

Balancing moments

(-22.95)x0.57=-13.1

(-22.95)x0.43=-9.9

Moment Distribution

	A	B		C
DF		0.57	0.43	
	M_{AB}	M_{BA}	M_{BC}	M_{CB}
FEM	3.75	-3.75	26.7	-26.7
Bal		-13.1	- 9.9	
CO	- 6.6			-5.0

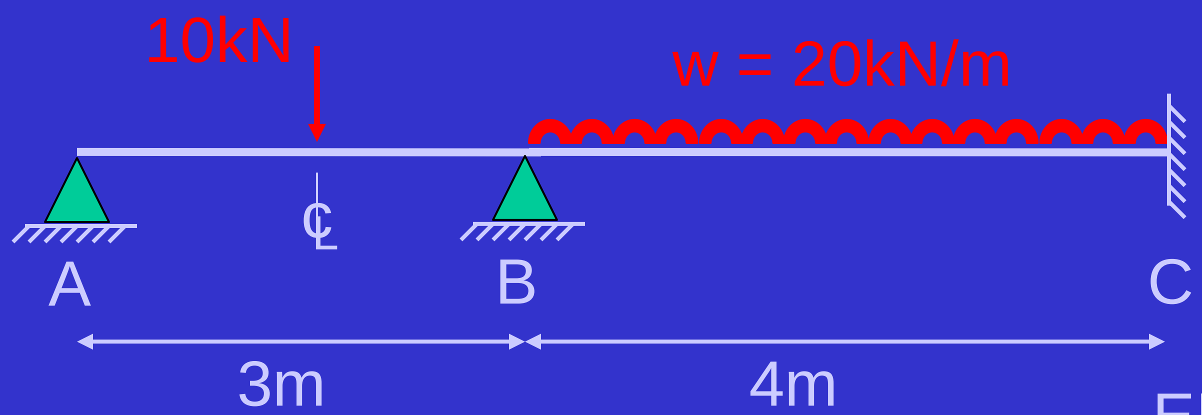
Carry over moment
(half of the balancing
moment)

Moment Distribution

	A		B		C
DF			0.57	0.43	
	M_{AB}		M_{BA}	M_{BC}	M_{CB}
FEM	3.75		-3.75	26.7	-26.7
Bal			-13.1	- 9.9	
CO	- 6.6				-5.0
Total	-2.9		-16.9	16.8	-31.7

Moment Distribution

So … what happens if we replace the fixed support at A with a pinned support ?



EI = constant

Fixed end moments:

M_{FAB} = PL/8 = 3.75 kNm M_{FBA} = - 3.75 kNm

M_{FBC} = $wL^2/12$ = 26.7 kNm M_{FBC} = - 26.7 kNm

Distribution factors:

$$DF_{BA} = \frac{1/3}{1/3 + 1/4} = 0.57 \qquad DF_{BC} = \frac{1/4}{1/3 + 1/4} = 0.43$$

Moment Distribution

	Pinned				Fixed
	A		B		C
DF	1	EI/L	0.57	0.43	EI/L	
	M_{AB}		M_{BA}	M_{BC}		M_{CB}
FEM	3.75		-3.75	26.7		-26.7

Moment Distribution

	Pinned					Fixed
	A		B			C
DF	1	EI/L	0.57	0.43	EI/L	
	M_{AB}		M_{BA}	M_{BC}		M_{CB}
FEM	3.75		-3.75	26.7		-26.7

Out of balance

Moment Distribution

	Pinned			Fixed
	A	B		C
DF	1	0.57	0.43	1
	M_{AB}	M_{BA}	M_{BC}	M_{CB}
FEM	3.75	-3.75	26.7	-26.7
Bal	-3.75	-13.1	-9.9	

Balancing moments

Moment Distribution

	Pinned			Fixed
	A	B		C
DF	1	0.57	0.43	1
	M_{AB}	M_{BA}	M_{BC}	M_{CB}
FEM	3.75	-3.75	26.7	-26.7
Bal	-3.75	-13.1	- 9.9	
CO	- 6.6	-1.9		- 5.0

Carry over moments

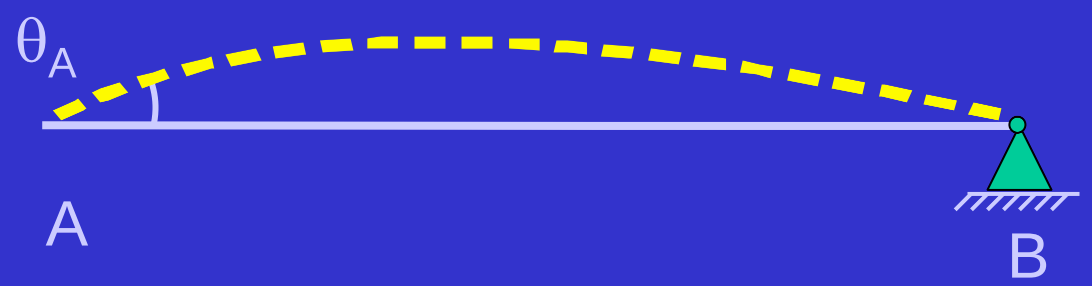
Moment Distribution

	Pinned A	B		Fixed C
DF	1	0.57	0.43	1
	M_{AB}	M_{BA}	M_{BC}	M_{CB}
FEM	3.75	-3.75	26.7	-26.7
Bal	-3.75	-13.1	- 9.9	
CO	- 6.6	-1.9		- 5.0
Bal	6.6	1.1	0.8	
CO	0.6	3.3		0.4
Bal	-0.6	- 1.9	- 1.4	
CO	- 1.0	-0.3		-0.7
Bal	1.0	0.2	0.1	
CO		0.5		
Bal		- 0.3	-0.2	
Total	0	- 16.2	16.1	-32.0

Moment Distribution

You can see that the pin end at A causes a very slow convergence. We can improve on this situation.

Consider a beam AB with a rotation at A and B is pinned.

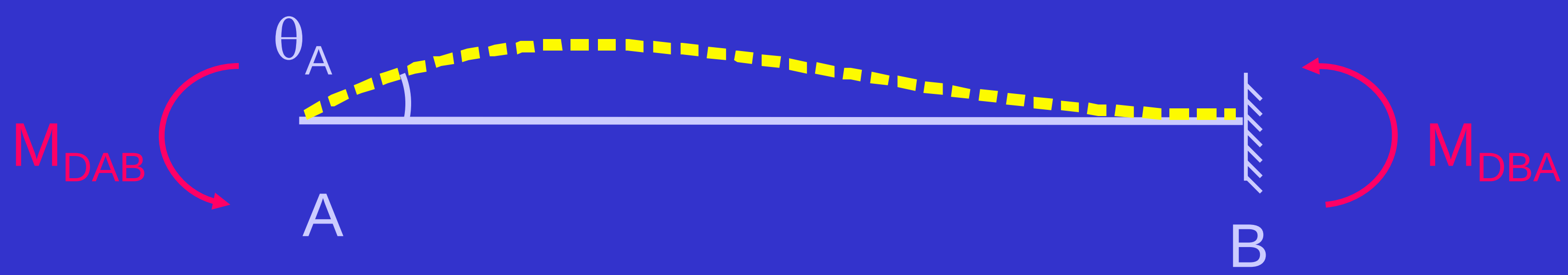


First we fully fix B and impose a rotation at A

Moment Distribution

You can see that the pin end at A causes a very slow convergence. We can improve on this situation.

Consider a beam AB with a rotation at A and B is pinned.



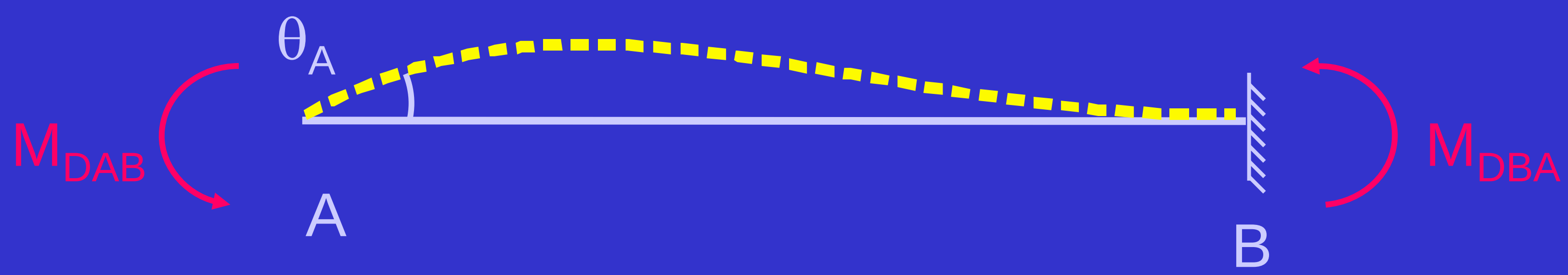
First we fully fix B and impose a rotation at A

$$M_{DAB} = \frac{4EI\theta_A}{L} \longrightarrow M_{DBA} = \frac{2EI\theta_A}{L}$$

Moment Distribution

You can see that the pin end at A causes a very slow convergence. We can improve on this situation.

Consider a beam AB with a rotation at A and B is pinned.



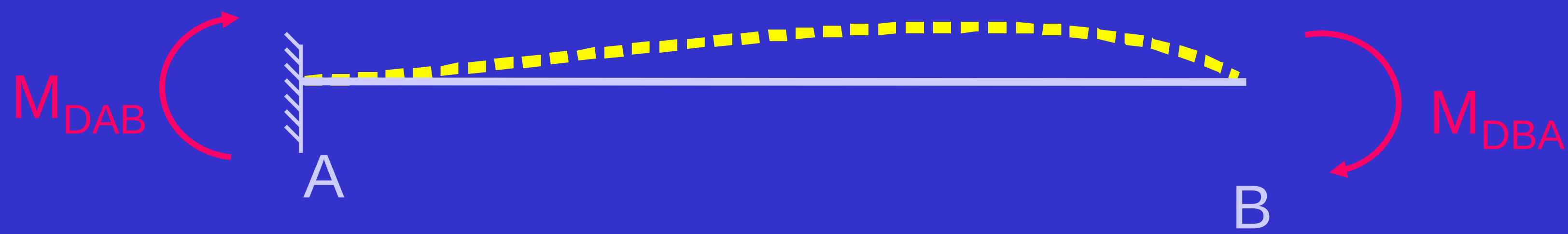
Now fully fix A and impose a balancing moment at B

$$M_{DAB} = \frac{4EI\theta_A}{L} \longrightarrow M_{DBA} = \frac{2EI\theta_A}{L}$$

Moment Distribution

You can see that the pin end at A causes a very slow convergence.

Consider a beam AB with a rotation at A and B is pinned.



Now fully fix A and impose a balancing moment at B

$$M_{DAB} = \frac{4EI\theta_A}{L} \qquad M_{DBA} = \frac{2EI\theta_A}{L}$$

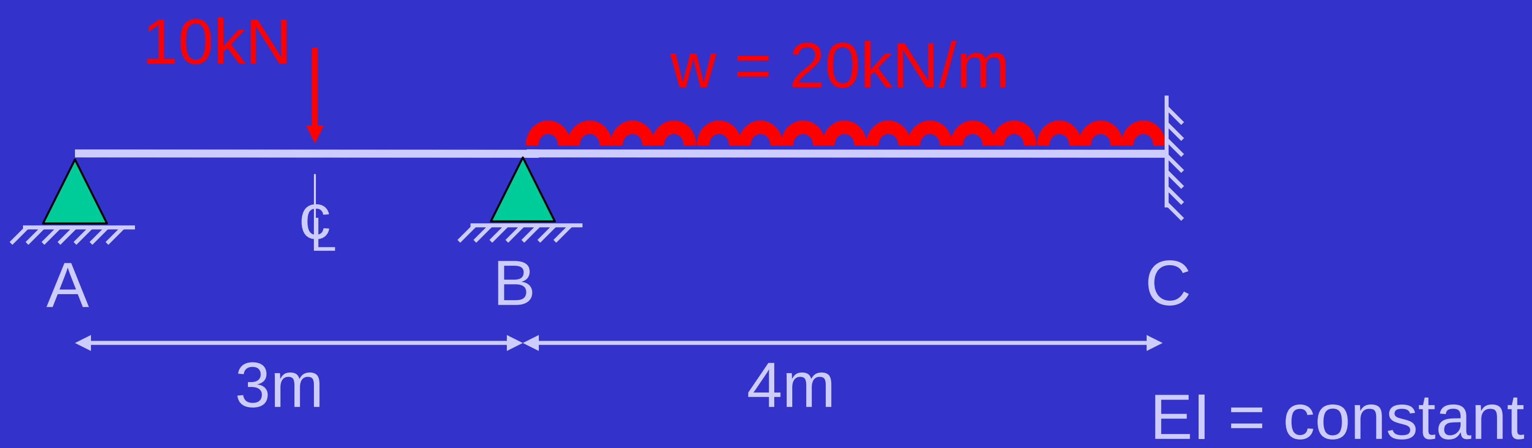
$$M_{DAB} = \frac{-EI\theta_A}{L} \qquad M_{DBA} = \frac{-2EI\theta_A}{L}$$

$$M_{DAB} = \frac{3EI\theta_A}{L} \qquad M_{DBA} = 0$$

Moment Distribution

So … if we replace the 4EI/L stiffness of a pin support member with 3EI/L and have a zero carry-over factor, the end result is the same.

Try this with the same problem as before:



Moment Distribution

Distribution factors:

$$DF_{BA} = \frac{3EI/L_{AB}}{3EI/L_{AB} + 4EI/L_{BC}} = \frac{3/(4\times 3)}{3/(4\times 3) + 1/4} = 0.5$$

$$DF_{Bc} = \frac{EI/L_{BC}}{3EI/4L_{AB} + EI/L_{BC}} = \frac{1/4}{3/(4\times 3) + 1/4} = 0.5$$

Moment Distribution

	Pinned					Fixed
	A		B			C
DF	1	0.75EI/L	0.5	0.5	EI/L	
	M_{AB}		M_{BA}	M_{BC}		M_{CB}
FEM	3.75		-3.75	26.7		-26.7
Bal	-3.75		-11.5	-11.5		
CO			- 1.9			- 5.6

No carry over

Moment Distribution

	Pinned					Fixed
	A		B			C
DF	1	0.75EI/L	0.5	0.5	EI/L	
	M_{AB}		M_{BA}	M_{BC}		M_{CB}
FEM	3.75		-3.75	26.7		-26.7
Bal	-3.75		-11.5	-11.5		
CO			- 1.9			- 5.6
Bal			1.0	1.0		
						0.5
Total	0		- 16.2	16.2		-31.8

This is the same result as before.

Moment Distribution

	Pinned			Fixed
	A	B		C
DF	1	0.57	0.43	1
	M_{AB}	M_{BA}	M_{BC}	M_{CB}
FEM	3.75	-3.75	26.7	-26.7
Bal	-3.75	-13.1	- 9.9	
CO	- 6.6	-1.9		- 5.0
Bal	6.6	1.1	0.8	
CO	0.6	3.3		0.4
Bal	-0.6	- 1.9	- 1.4	
CO	- 1.0	-0.3		-0.7
Bal	1.0	0.2	0.1	
CO		0.5		
Bal		- 0.3	-0.2	
Total	0	- 16.2	16.1	-32.0

Moment Distribution