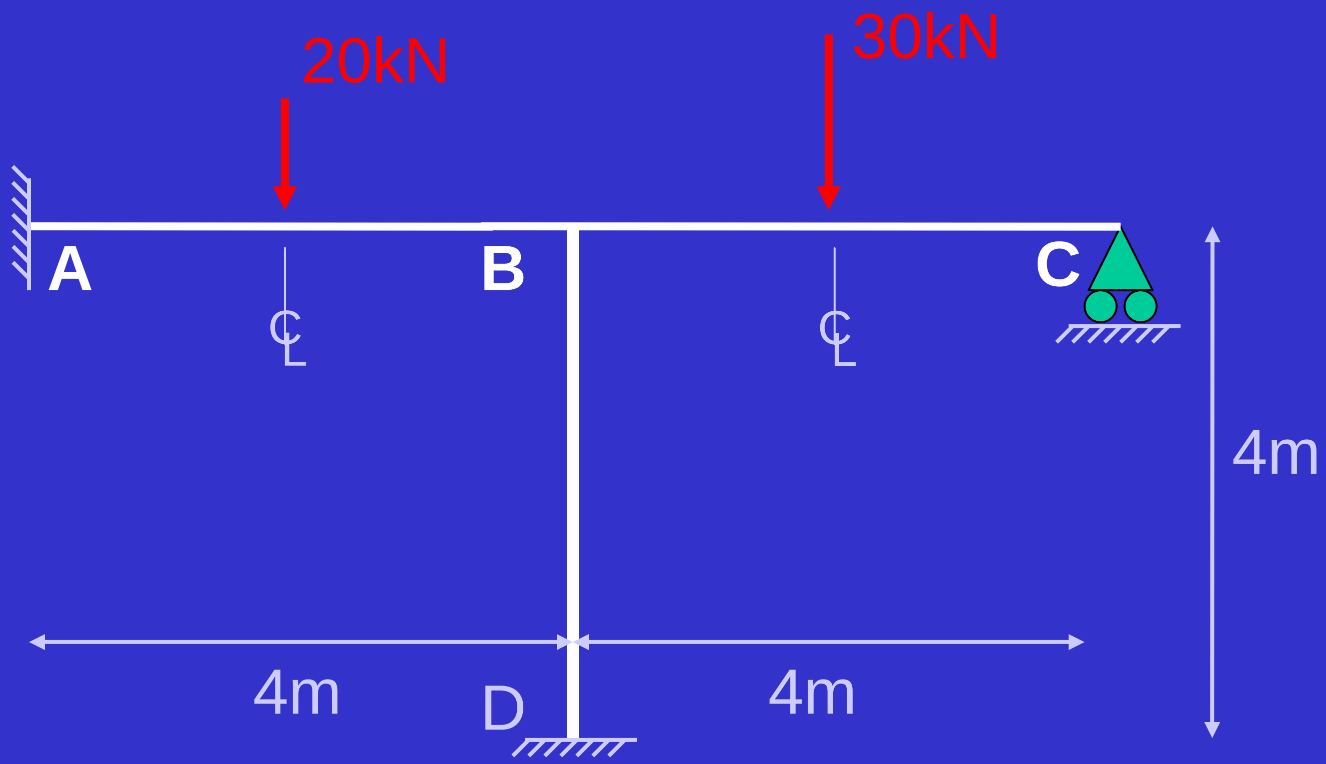
Moment Distribution - Frames



Consider a 3 beam frame

Joints A & D are fixed. Joint C (& B) can rotate but not deflect.

The principal deflections are due to bending, i.e. ignore deflections due to axial deformation of BD.

Moment Distribution - Frames

Fixed end moments:

M_{FAB} = 20 x 4 / 8 = 10 kN/m M_{FBA} = -10 kN/m

M_{FBC} = 30 x 4 / 8 = 15 kN/m M_{FCB} = -15 kN/m

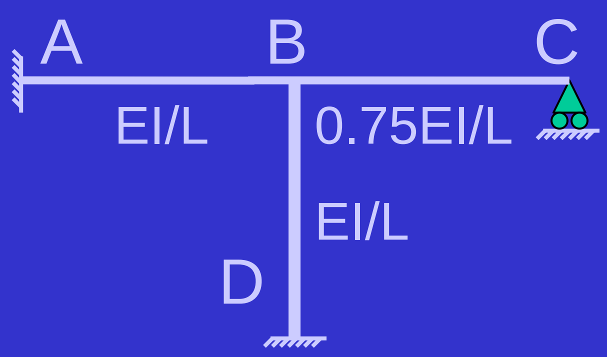
Distribution factors:

$Stiffness_{AB}$ = EI/L $Stiffness_{BD}$ = EI/L $Stiffness_{BC}$ = 0.75EI/L

D_{FBA} = 1 / (1 + 1 + 0.75) = 0.364

D_{FBD} = 1 / (1 + 1 + 0.75) = 0.364

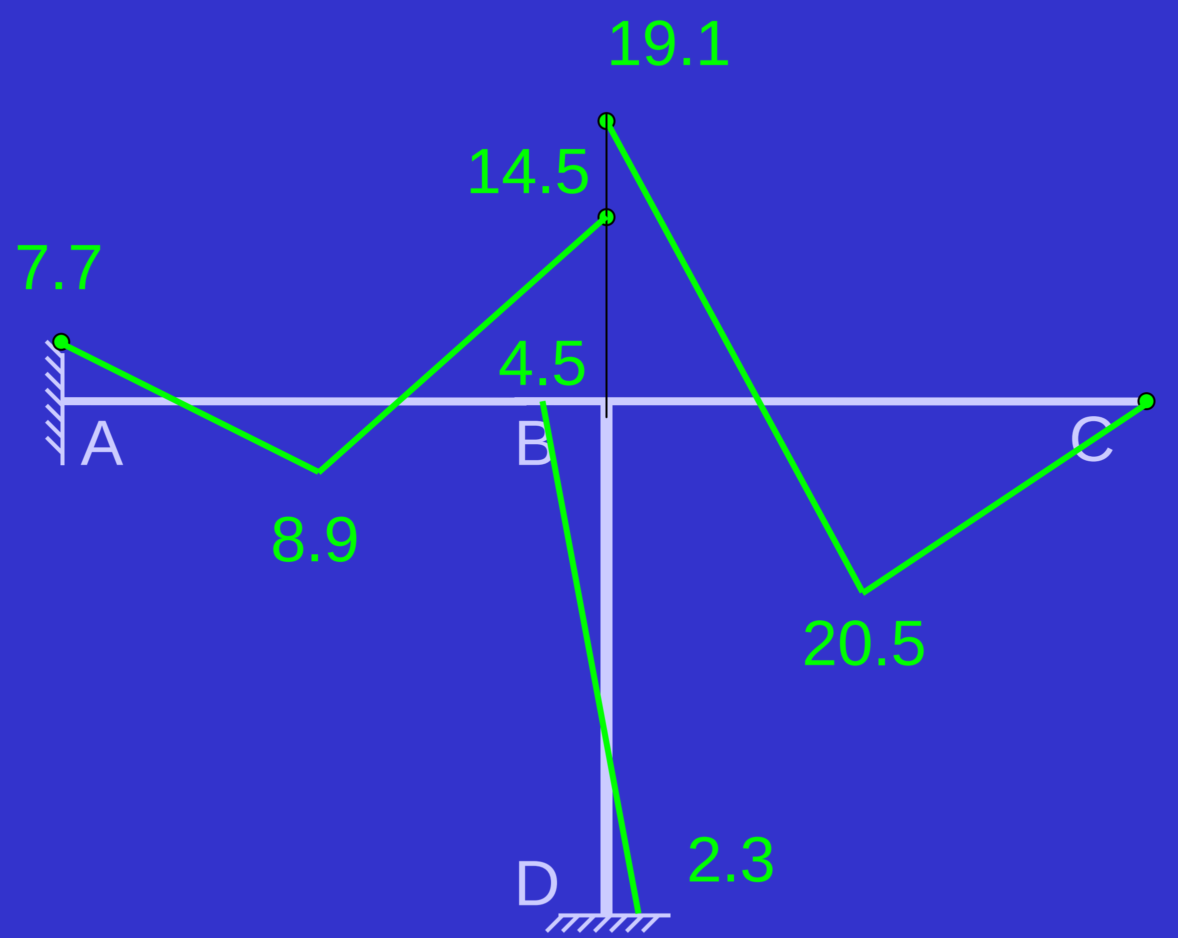
D_{FBC} = 0.75 / (1 + 1 + 0.75) = 0.273



Moment Distribution - Frames

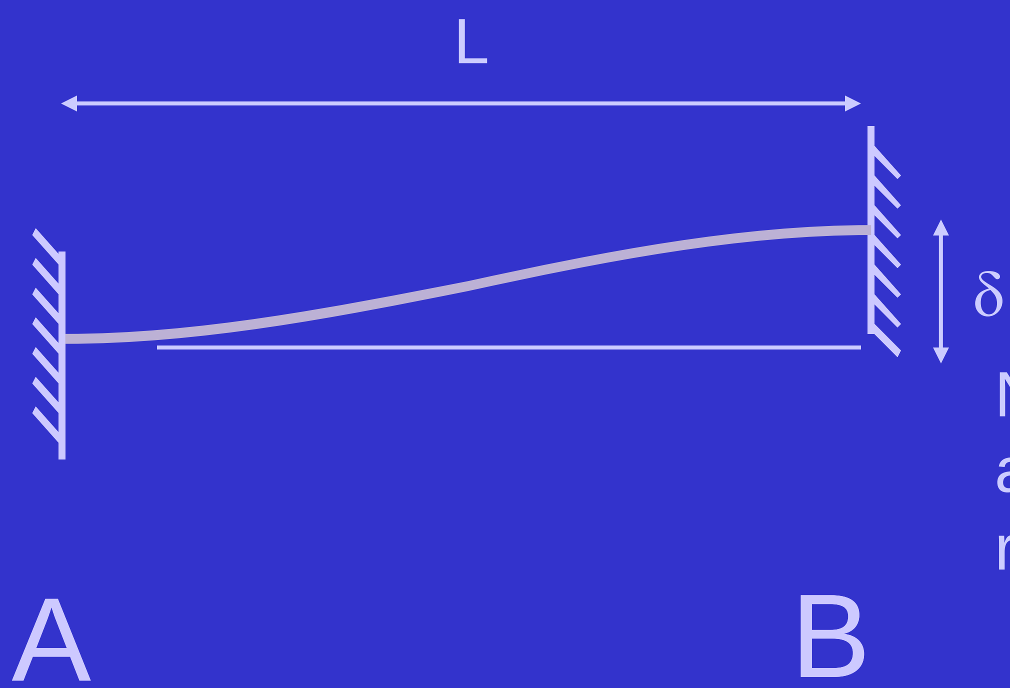
	Fixed A		B		Pinned C	Fixed D
		EI/L	EI/L	0.75EI/L		
	M_{AB}	M_{BA}	M_{BD}	M_{BC}	M_{CB}	M_{DB}
DF		0.364	0.364	0.273	1	
FEM	10	-10		15	-15	
Bal		-1.8	-1.8	-1.37	15	
CO	- 0.9			7.5		- 0.9
Bal		-2.7	-2.7	-2.04		
CO	- 1.4					- 1.4
Total	7.7	- 14.5	-4.5	19.1	0	-2.3

Moment Distribution - Frames



Moment Distribution - Frames

Now consider … what happens if support A is allowed to sink ?



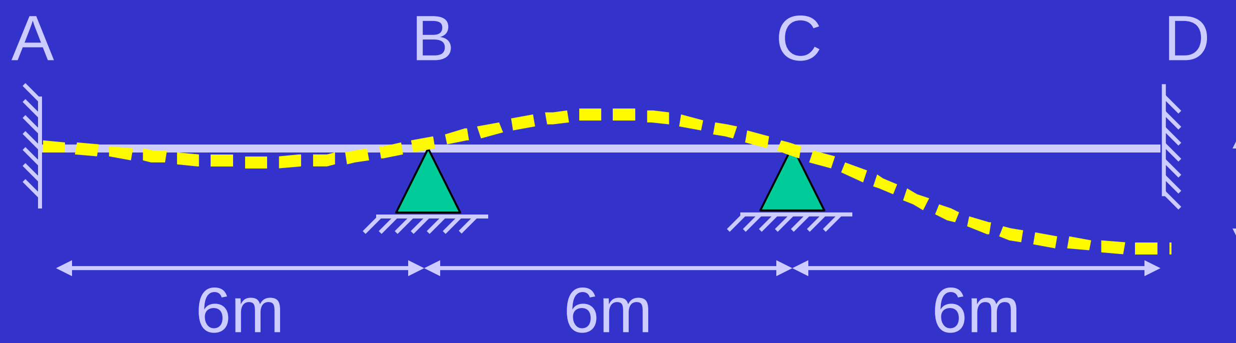
Note: δ causes anti-clockwise rotation → +ve

$$M_{DAB} = -\frac{6EI\delta}{L^2}$$

$$M_{DBA} = -\frac{6EI\delta}{L^2}$$

Moment Distribution - Frames

Now let's apply a sinking support to a three-span beam



$EI = 25 \times 10^3$ kNm^2

No loads, but support D sinks by 30 mm

-ve δ because rotation is clockwise

Fixed end moments:

$M_{FAB} = M_{FBA} = M_{FBC} = M_{FCB} = 0$

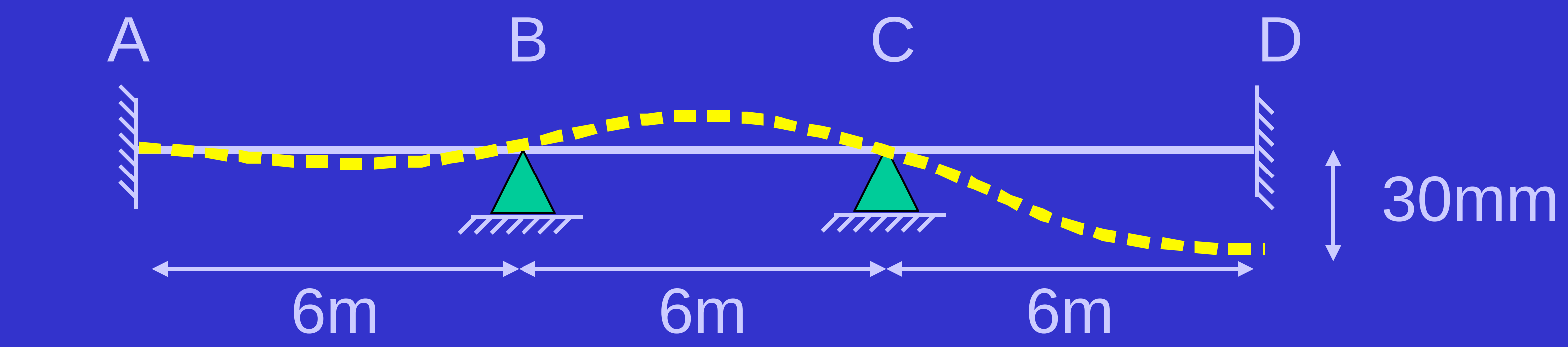
$$M_{DCD} = M_{DDC} = -\frac{6 \times 25 \times 10^3 \times (-0.03)}{6^2} = 125 \text{ kNm}$$

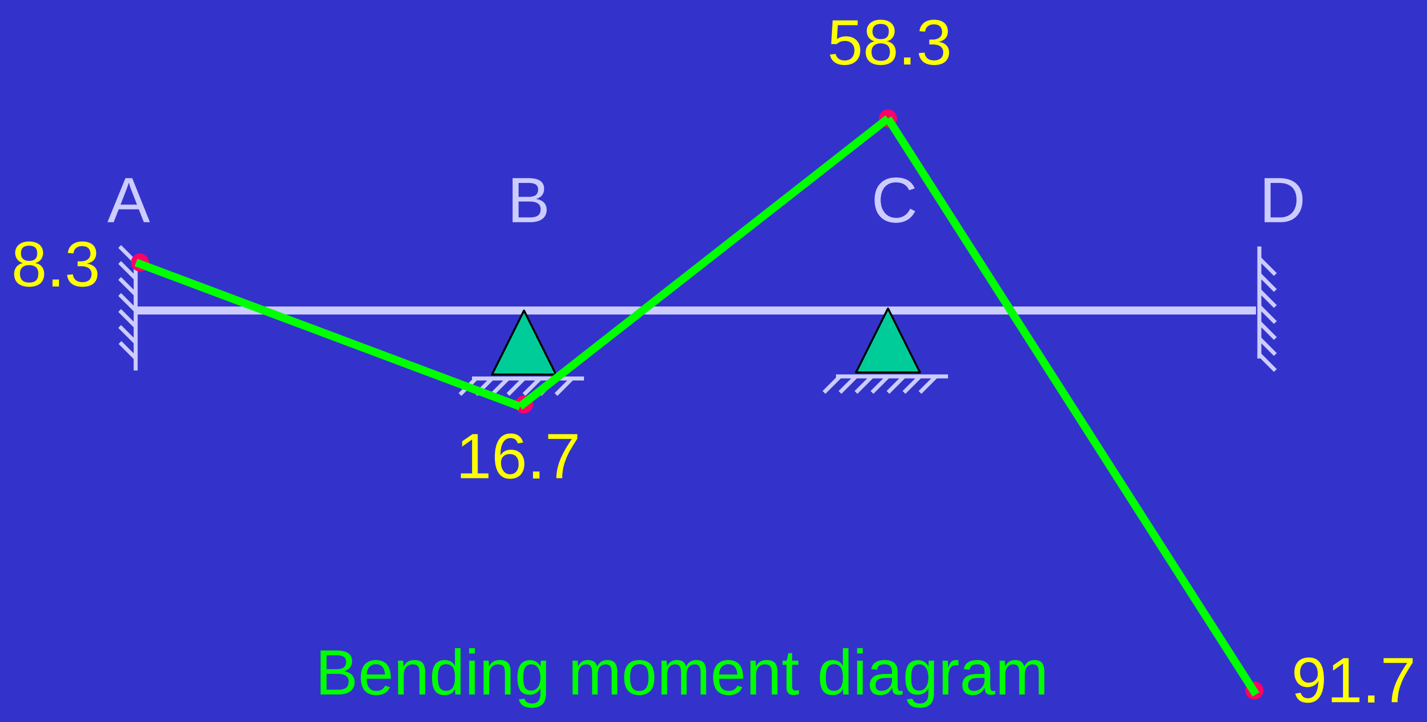
Moment Distribution - Frames

Fixed A — EI/L — B — EI/L — C — EI/L — D Fixed

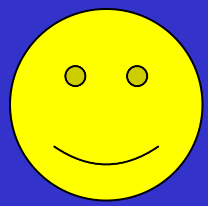
	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
DF	0	0.5	0.5	0.5	0.5	0
FEM					125	125
BAL				-62.5	-62.5	
CO			- 31.3			- 31.3
BAL		15.7	15.7			
CO	7.8			7.8		
BAL				-3.9	-3.9	
CO			-2.0			-2.0
BAL		1.0	1.0			
CO	0.5			0.5		
BAL				-0.3	-0.3	
Total	8.3	16.7	-16.6	-58.4	58.3	91.7

Moment Distribution - Frames





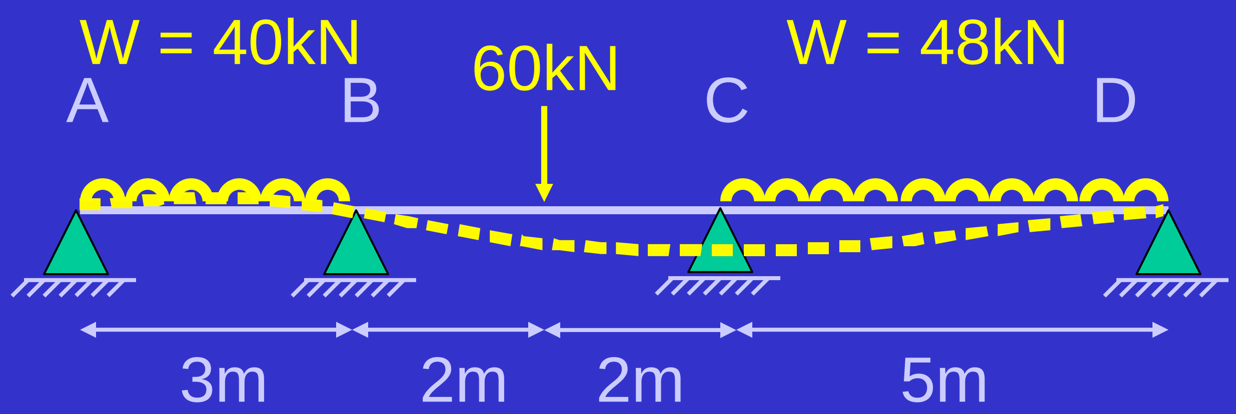
Moment Distribution - Frames



….more to come

Moment Distribution - Frames

Now let's apply a sinking support to a loaded three-span beam



EI = 20 x 10^3 kNm^2

Support C sinks by 4 mm

Loads on all 3 spans

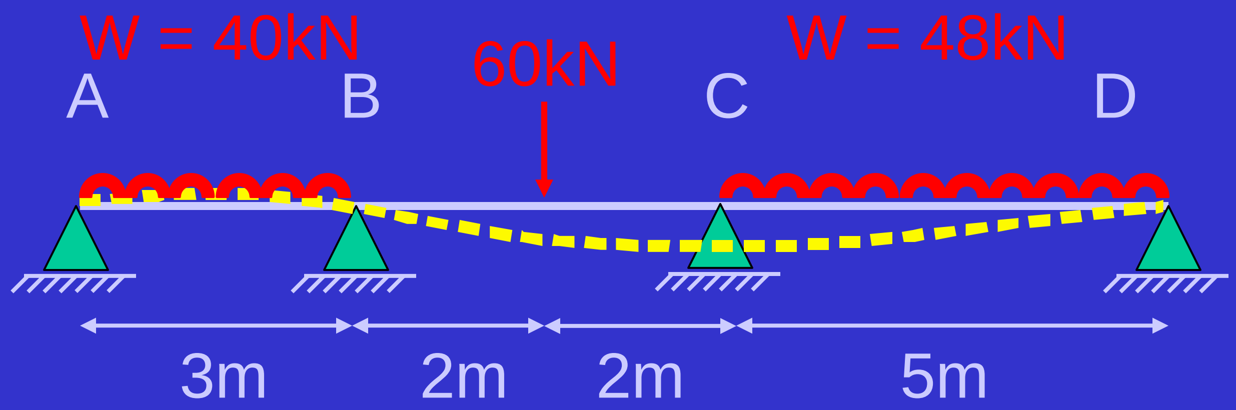
Method 1: use Principle of Superposition

- analyse sinking support
- analyse loaded beams

Add solutions together

Moment Distribution - Frames

Now let's apply a sinking support to a loaded three-span beam



EI = 20 x 10^3 kNm^2

Support C sinks by 4 mm

Loads on all 3 spans

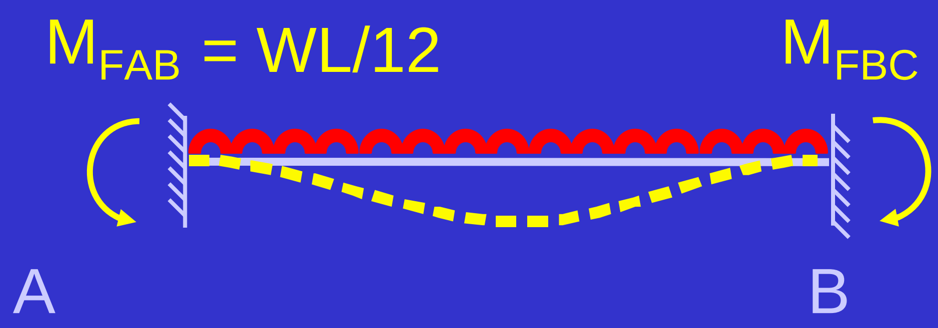
Method 2: Add all FEMs together & analyse in one go

Moment Distribution - Frames

Fixed end moments

No settlement:

M_{FAB} = WL/12 = 40 X 3 /12 = 10 kNm = - M_{FBA}



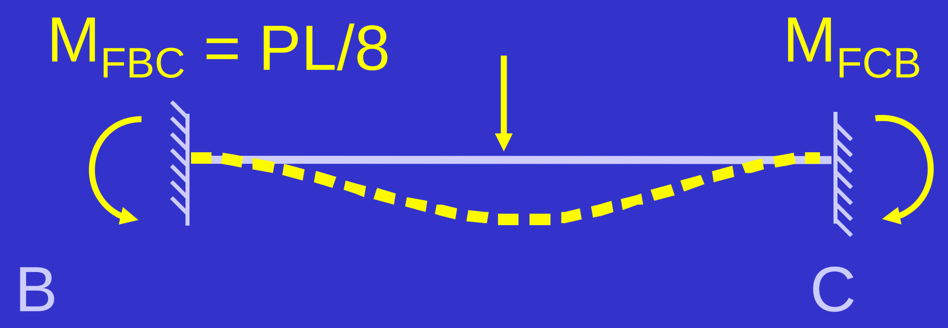
Moment Distribution - Frames

Fixed end moments

No settlement:

M_{FAB} = WL/12 = 40 X 3 /12 = 10 kNm = - M_{FBA}

M_{FBC} = PL/8 = 60 X 4 /8 = 30 kNm = - M_{FCB}



Moment Distribution - Frames

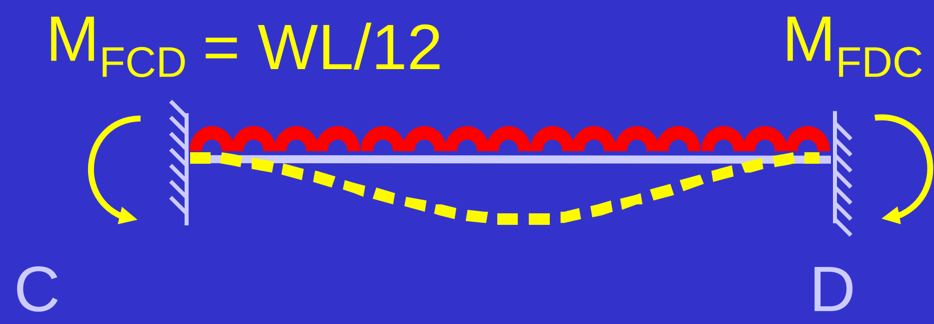
Fixed end moments

No settlement:

M_{FAB} = WL/12 = 40 X 3 /12 = 10 kNm = - M_{FBA}

M_{FBC} = PL/8 = 60 X 4 /8 = 30 kNm = - M_{FCB}

M_{FCD} = WL/12 = 48 X 5 /12 = 20 kNm = - M_{FDC}



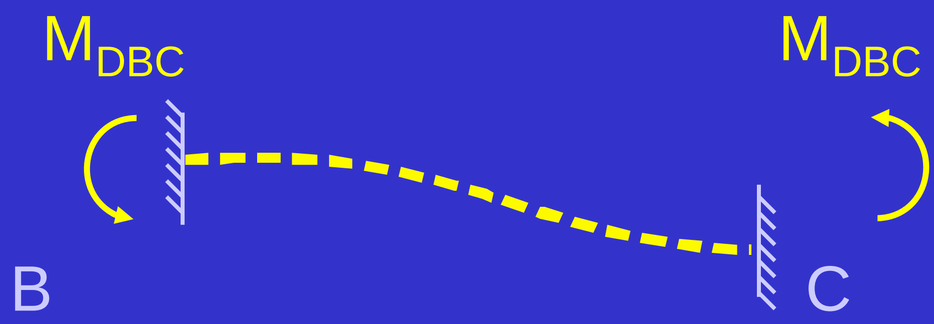
Moment Distribution - Frames

Fixed end moments

Settlement:

$$M_{DBC} = -\frac{6EI\delta}{L^2}$$

But … δ is -ve (clockwise rotation of BC)



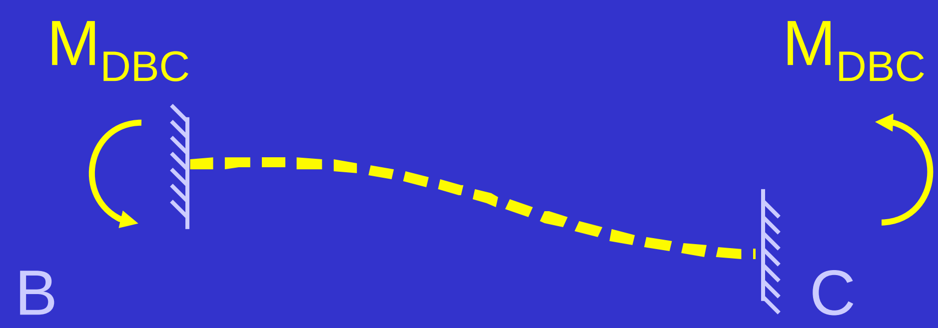
Moment Distribution - Frames

Fixed end moments

Settlement:

$M_{DBC} = -6 \times 20 \times 10^3 \times (-0.004) / 4^2 = 30 \text{ kNm} = M_{DCB}$

Same sign !!



Moment Distribution - Frames

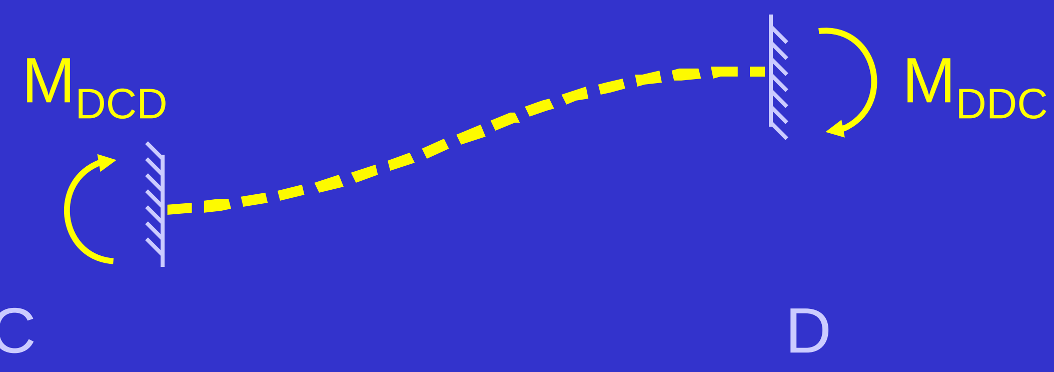
Fixed end moments

Settlement:

M_{DBC} = -6 X 20 X 10^3 X (-0.004) / 4^2 = 30 kNm = M_{DCB}

$$M_{DCD} = -\frac{6EI\delta}{L^2}$$

δ is +ve (anti-clockwise rotation of CD)



Moment Distribution - Frames

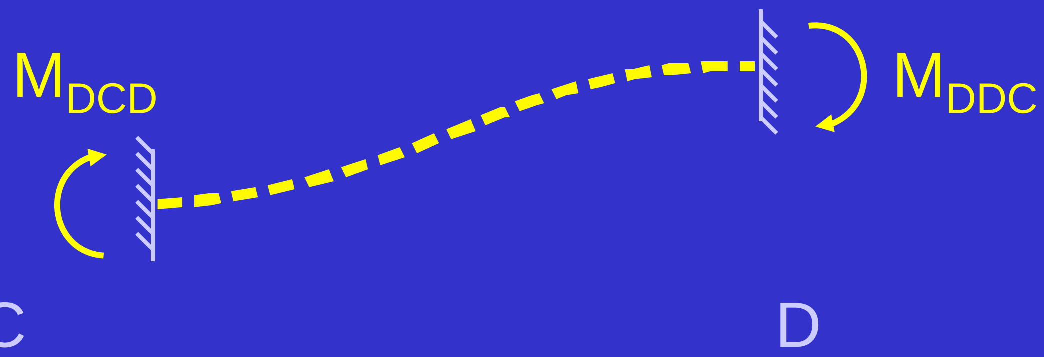
Fixed end moments

Settlement:

M_{DBC} = -6 X 20 X 10^3 X (-0.004) / 4^2 = 30 kNm = M_{DCB}

M_{DCD} = -6 X 20 X 10^3 X (0.004) / 5^2 = -19.2 kNm = M_{DDC}

Same sign !!



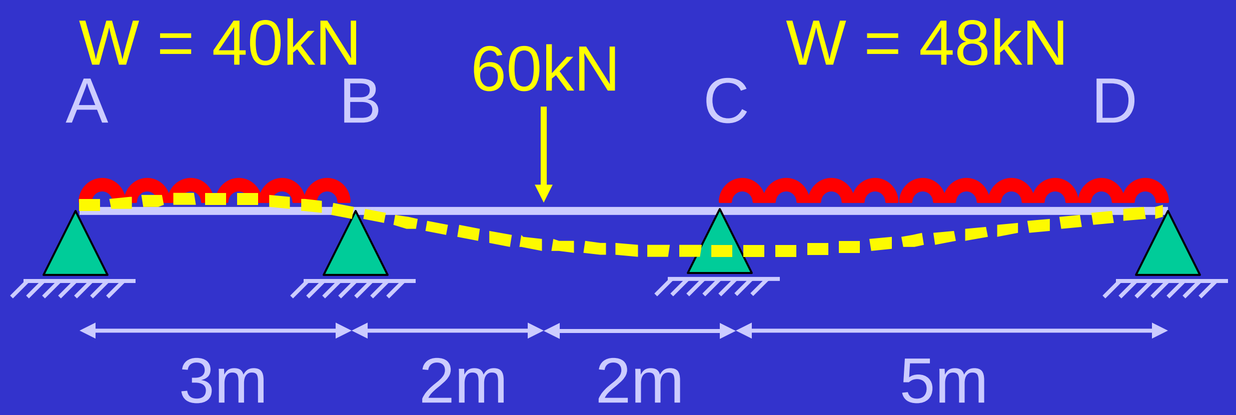
Moment Distribution - Frames

Pinned A — EI/4 — B — EI/4 — C — 0.15EI — D Pinned

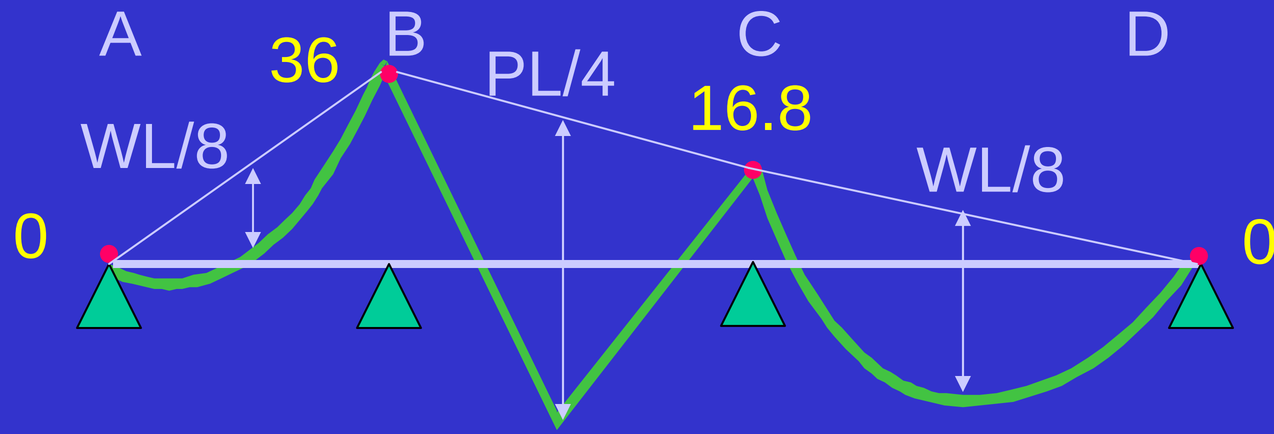
A and D are pinned.

	M_{AB}	M_{BA}	M_{BC}	M_{CB}	M_{CD}	M_{DC}
DF	1	0.5	0.5	0.625	0.375	1
FEM			30	30	-19	-19
FEM	10	-10	30	-30	20	-20
BAL	-10	-25	-25	-0.6	-0.4	39
CO	No c/o	-5	-0.3	-12.5	19.5	No c/o
BAL		2.6	2.6	-4.5	-2.6	
CO			-2.3	1.3		
BAL		1.2	1.2	-0.8	-0.5	
CO			-0.4	0.6		
BAL		0.2	0.2	-0.3	-0.2	
Total	0	-36.0	36.1	-16.7	16.8	0

Moment Distribution - Frames

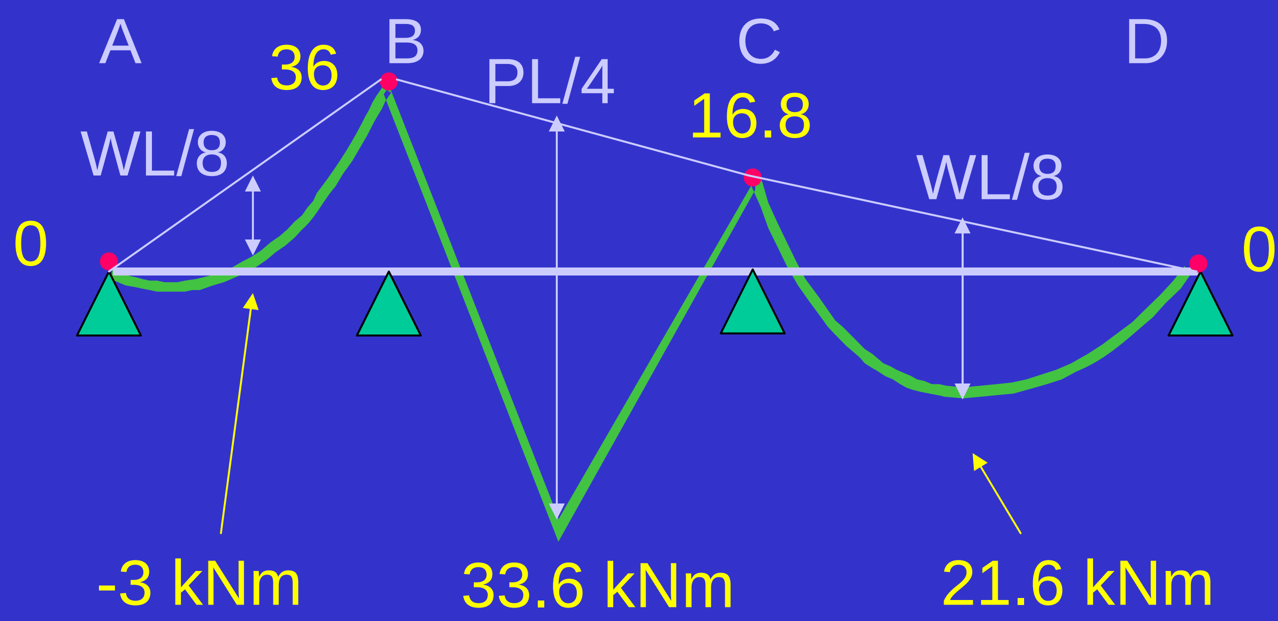


Support C sinks by 4 mm



Bending moment diagram

Moment Distribution - Frames

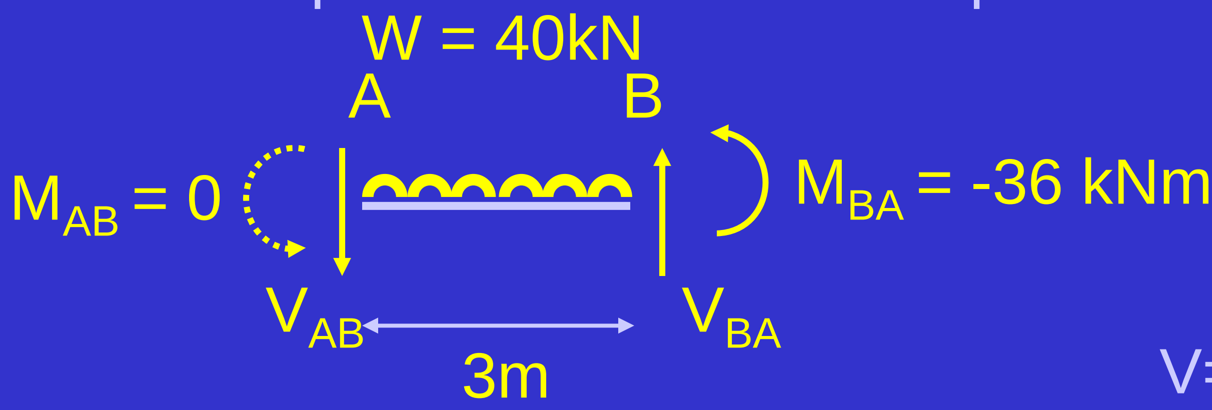


Mid-span moments (sagging +ve)

…but midspan moments are not necessarily the maximum span moments !!

Moment Distribution - Frames

Look at equilibrium of individual spans



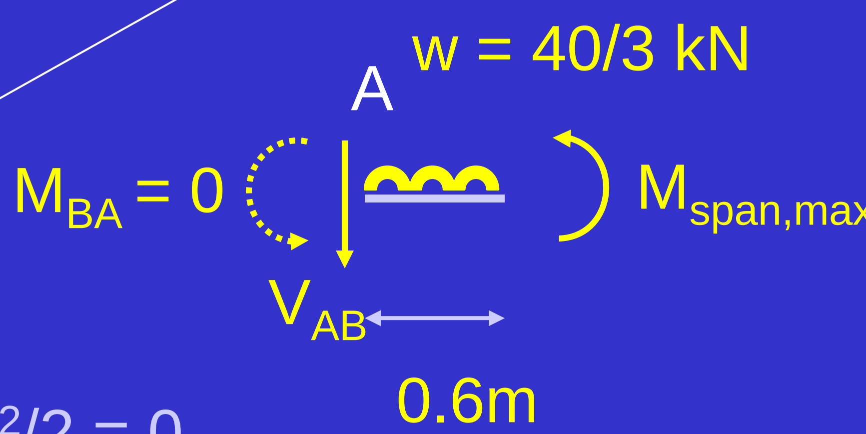
$V = V_{AB} + (40/3)x = 0$

$V_{AB} = -W/2 - (M_{AB} + M_{BA})/L$

$V_{AB} = -40/2 + 36/3 \quad = -8 \text{ kN}$

$M_{span,max}$ @ zero shear

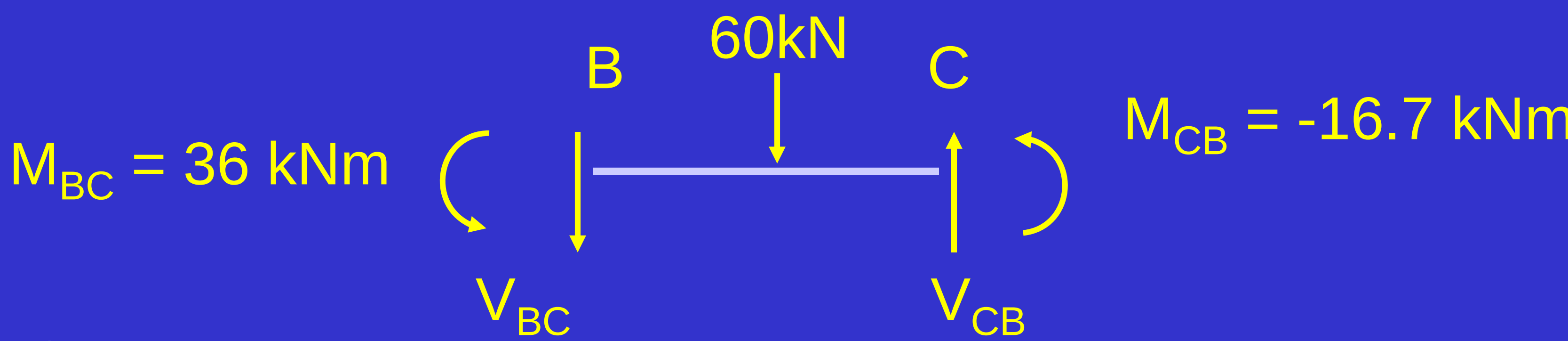
$x = 8 / (40/3) = 0.6\text{m}$



$M_{span,max}$ $-8 \times 0.6 + (40/3) \times 0.6^2/2 = 0$

$M_{span,max} = 2.4 \text{ kNm (sagging)}$

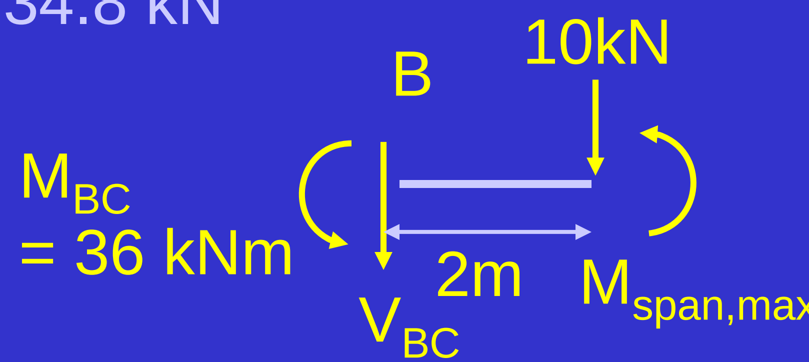
Moment Distribution - Frames



$V_{BC} = -P/2 - (M_{BC} + M_{CB})/L$

$V_{BC} = -60/2 - (36 - 16.7)/4 \quad = -34.8$ kN

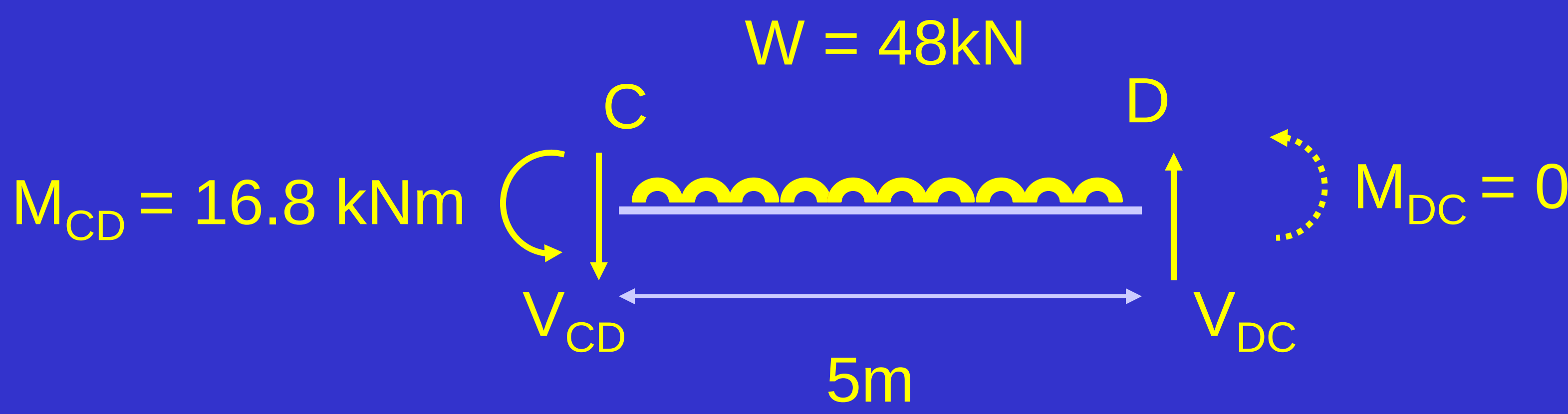
Hence zero shear @ ℄ , x = 2



$M_{span,max} - 34.8 \times 2 + 36 = 0$

$M_{span,max} = 33.6$ kNm (sagging)

Moment Distribution - Frames

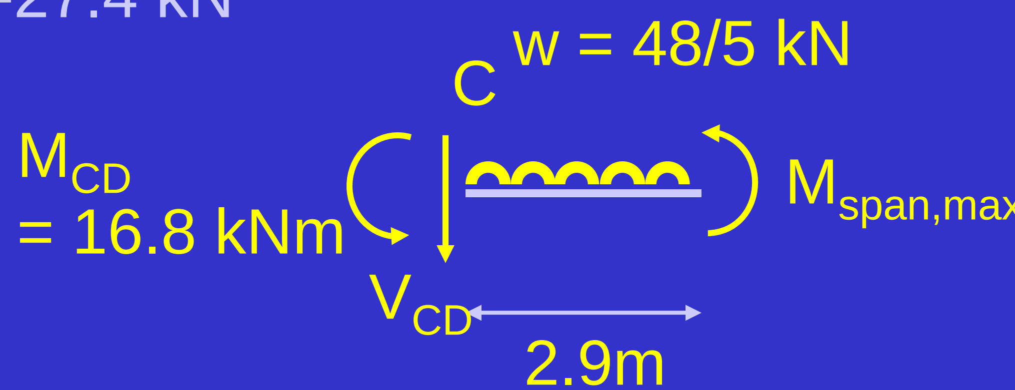


V_{CD} = - W/2 - (M_{CD} + M_{DC})/L

V_{CD} = - 48/2 - 16.8/5 = -27.4 kN

$M_{span,max}$ @ zero shear

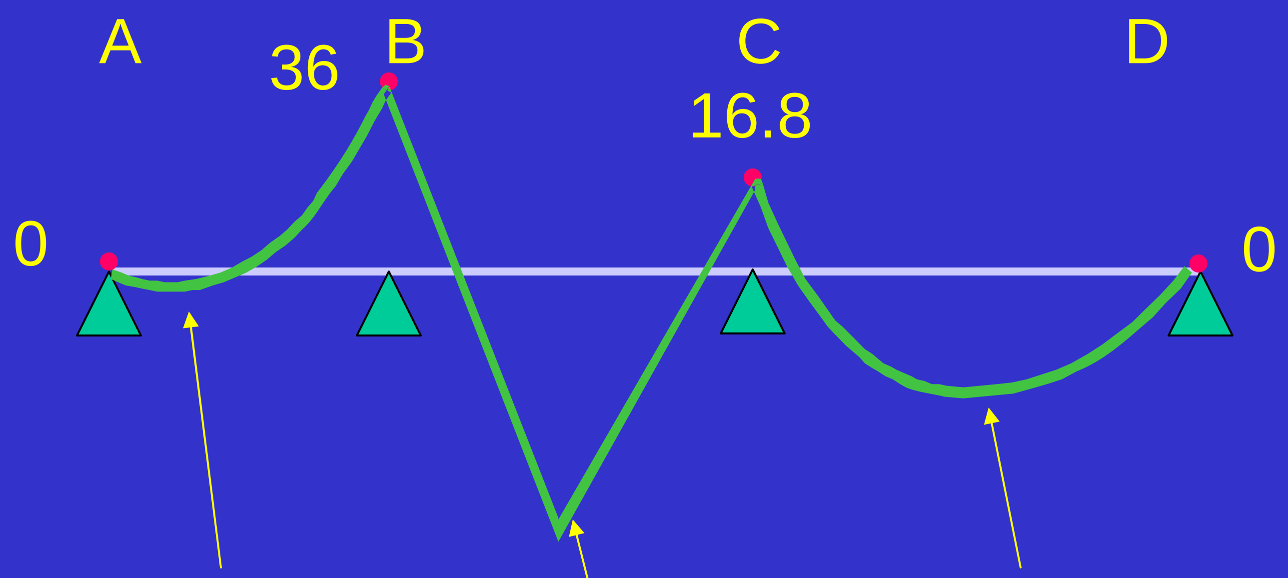
x = 27.4 / (48/5) = 2.9m



$M_{span,max}$ + 16.8 -27.4 x 2.9 + (48/5) x 2.9^2/2 = 0

$M_{span,max}$ = 22.3 kNm (sagging)

Moment Distribution - Frames



-3 kNm 33.6 kNm 21.6 kNm

2.4 kNm 33.6 kNm 24.6 kNm

Mid-span moments (sagging +ve)

Maximum span moments (sagging +ve)

Moment Distribution - Frames