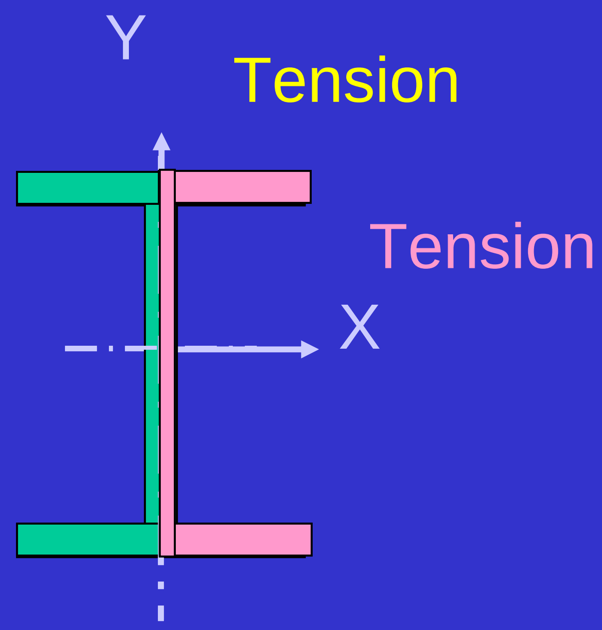
Biaxial bending of asymmetric section



Define +ve M_x as causing +ve tensile stresses when y is +ve

Define +ve M_y as causing +ve tensile stresses when x is +ve

Biaxial bending of asymmetric section

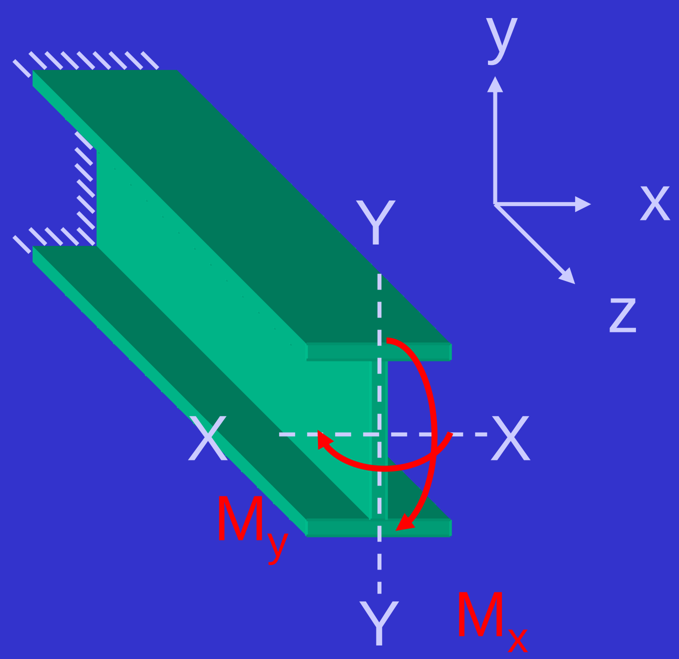
What happens if we try and bend an asymmetric section without applying the load at the shear centre?

For the case of biaxial bending of symmetric section:

+ve M_y gives +ve tension on +ve X side of section

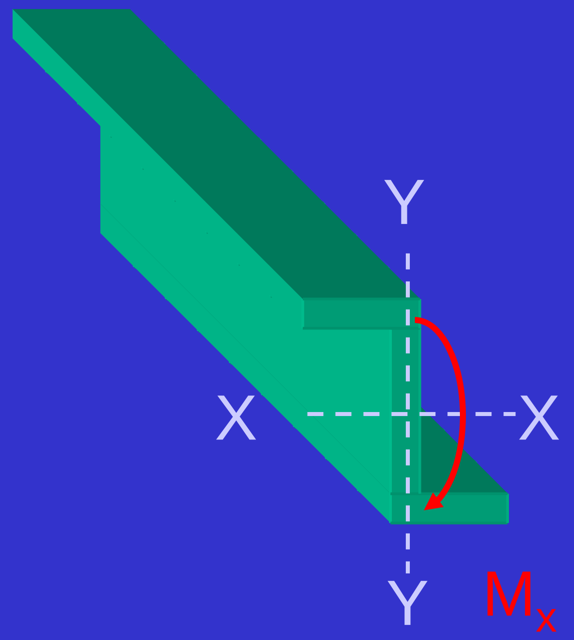
$$\sigma_z = \frac{M_x\, y}{I_{xx}} + \frac{M_y\, x}{I_{yy}}$$

+ve M_x gives +ve tension on +ve Y side of section

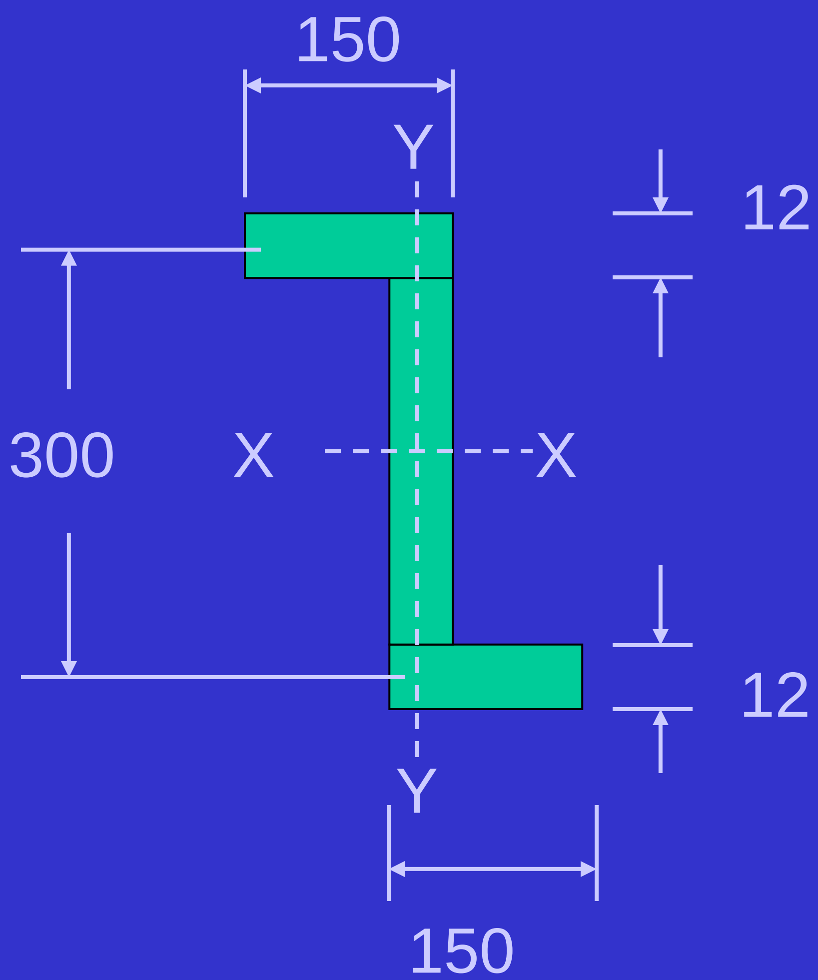


Biaxial bending of asymmetric section

What happens if we try and apply the same equation to an asymmetric section for a moment M_x ?



Biaxial bending of asymmetric section



$I_{xx} = 95.7 \times 10^6\ mm^4$

$I_{yy} = 23.9 \times 10^6\ mm^4$

Now apply a hogging (positive) bending moment of

$M_x = 10\ kN.m = 10 \times 10^6\ N.mm$

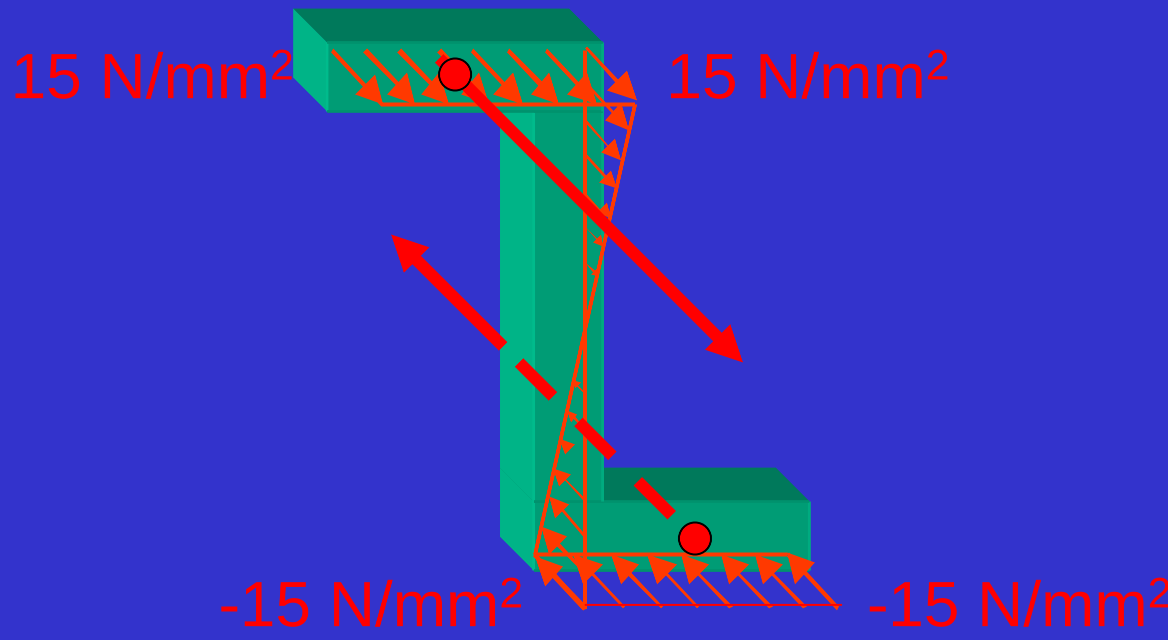
Biaxial bending of asymmetric section

$$\sigma_z = \frac{M_x y}{I_{xx}} + \frac{M_y x}{I_{yy}} \qquad = 0.105\, y\ \text{N.mm}^{-3}$$

The average stress in the top flange at y = 144 mm
is σ_z = 15 N/mm^2 (tension positive)

Similarly the average stress in the bottom flange at
y = -144 mm
is σ_z = -15 N/mm^2 (compression negative)

Biaxial bending of asymmetric section



The forces in the flanges result in horizontal forces that are laterally out of alignment.

This result in a lateral bending moment about the Y-Y axis… and yet there is no external M_y moment applied !

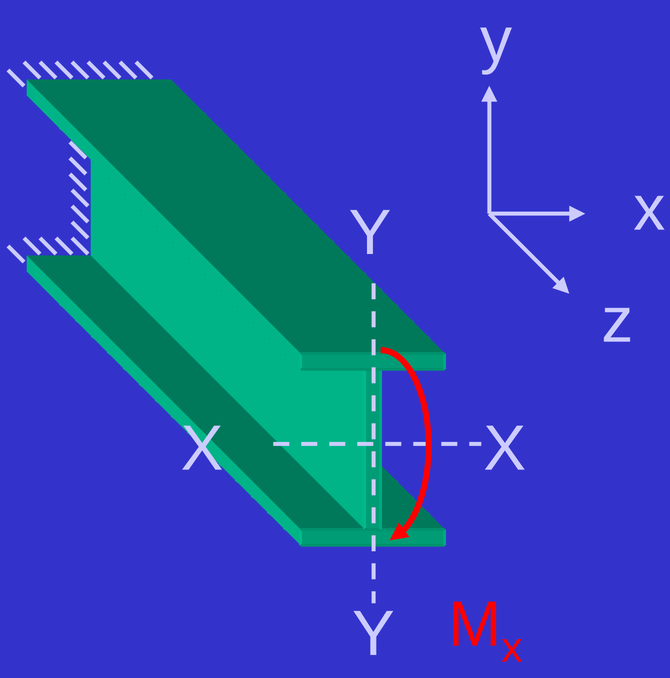
Although only M_x is applied, the section must bend about Y-Y

Biaxial bending of asymmetric section

General expression for bending stress

For simple bending M_x of a section symmetric about the Y-Y axis, axial displacement at any section (in the Z direction) is given by:

$$u = C\,y$$



and the axial strain is given by:

$$\varepsilon_z = \frac{du}{dz}$$

Biaxial bending of asymmetric section

General expression for bending stress

For an asymmetric section, we must replace this function for u by a more general expression:

$$u = C_1 y + C_2 x$$

$$now\ \varepsilon_z = \frac{du}{dz} = C_1' y + C_2' x$$

$$where\ C_1' = {dC_1}/{dz} \quad and\ C_2' = {dC_2}/{dz}$$

$$Note: \sigma_z = E\varepsilon_z$$

$$hence\ \boxed{\sigma_z = E.C_1'.y + E.C_2'.x}$$

Biaxial bending of asymmetric section

Moment about the x axis = M_x

$$M_x = \int \sigma_z y \, dA \quad = EC_1' \int y^2 \, dA + EC_2' \int xy \, dA$$

Moment about the y axis = M_y

$$M_y = \int \sigma_z x \, dA \quad = EC_1' \int xy \, dA + EC_2' \int x^2 \, dA$$

Biaxial bending of asymmetric section

Define :

$$\int y^2 \, dA = I_{xx}$$ = second moment of area about the x axis

$$\int x^2 \, dA = I_{yy}$$ = second moment of area about the y axis

$$\int xy \, dA = I_{xy}$$ = cross second moment of area about the x-y axes

I_{xy} is also known as the product moment of area

Biaxial bending of asymmetric section

Hence:

$$M_x = EC_1' I_{xx} + EC_2' I_{xy}$$

$$M_y = EC_1' I_{xy} + EC_2' I_{yy}$$

Solving for C_1' and C_2' gives:

$$EC_1' = \frac{\left(M_x I_{yy} - M_y I_{xy}\right)}{\left(I_{yy} I_{xx} - I_{xy}{}^2\right)}$$

$$EC_2' = \frac{\left(-M_x I_{xy} + M_y I_{xx}\right)}{\left(I_{yy} I_{xx} - I_{xy}{}^2\right)}$$

Biaxial bending of asymmetric section

$$\sigma_z = EC_1'y + EC_2'x$$

$$\sigma_z = \frac{\left(M_x I_{yy} - M_y I_{xy}\right)}{\left(I_{yy} I_{xx} - I_{xy}{}^2\right)} y + \frac{\left(-M_x I_{xy} + M_y I_{xx}\right)}{\left(I_{yy} I_{xx} - I_{xy}{}^2\right)} x$$

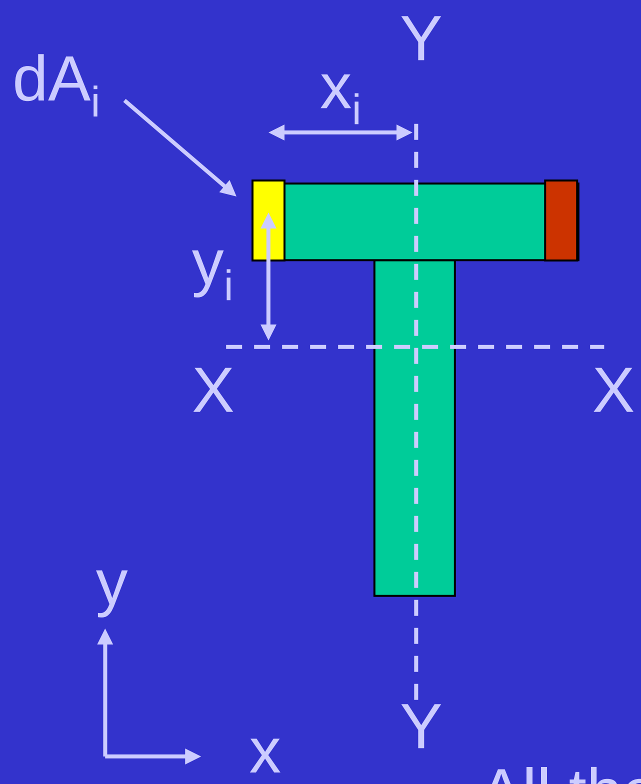
Note that if $I_{xy} = 0$,

$$\sigma_z = \frac{M_x}{I_{xx}} y + \frac{M_y}{I_{yy}} x$$

This is the same formula as used for a symmetric section

Biaxial bending of asymmetric section

Why is $I_{xy} = 0$ for a symmetric section ?



$$I_{xy} = \int xy\, dA$$

$$I_{xy} = \sum x_i y_i dA_i$$

For every area dA_i with location +ve y, -ve x there is a corresponding area dA_i with location +ve y, +ve x

All the contributions to $\Sigma x_i y_i dA_i$ will cancel out

Biaxial bending of asymmetric section

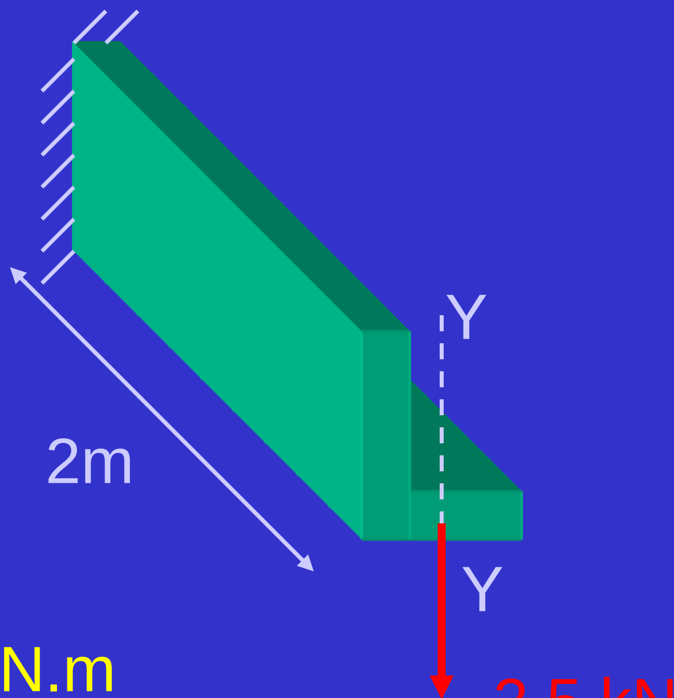
For a section with at least one axis of symmetry about X-X or Y-Y:

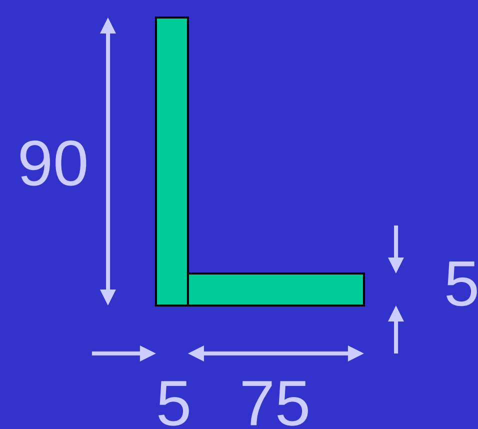
$$I_{xy} = 0$$

Biaxial bending of asymmetric section

Example:

The L-section cantilever shown has a point load of 2.5 kN applied at the tip of the cantilever. The load is applied at the centroidal Y-Y axis. What are the bending stresses ?

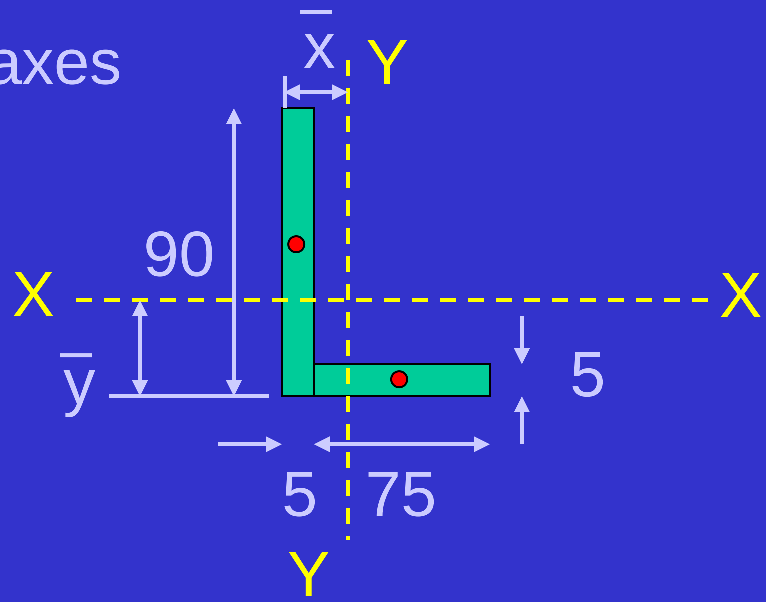




M_x = 5.0 kN.m
(hogging +ve)

Biaxial bending of asymmetric section

Find the centroidal axes



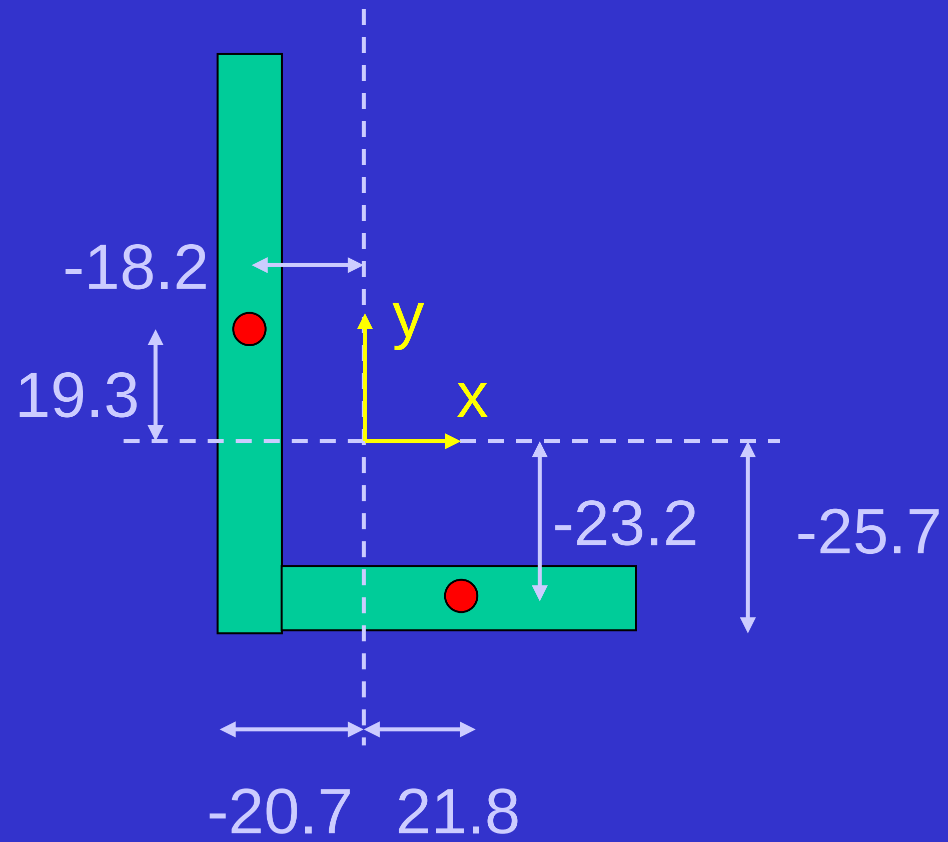
Total area = 90 x 5 + 75 x 5 = 825 mm^2

$$\bar{y} = \frac{(90\times5\times45)+(75\times5\times2.5)}{825} = 25.7 \text{ mm}$$

$$\bar{x} = \frac{(90\times5\times2.5)+(75\times5\times42.5)}{825} = 20.7 \text{ mm}$$

Biaxial bending of asymmetric section

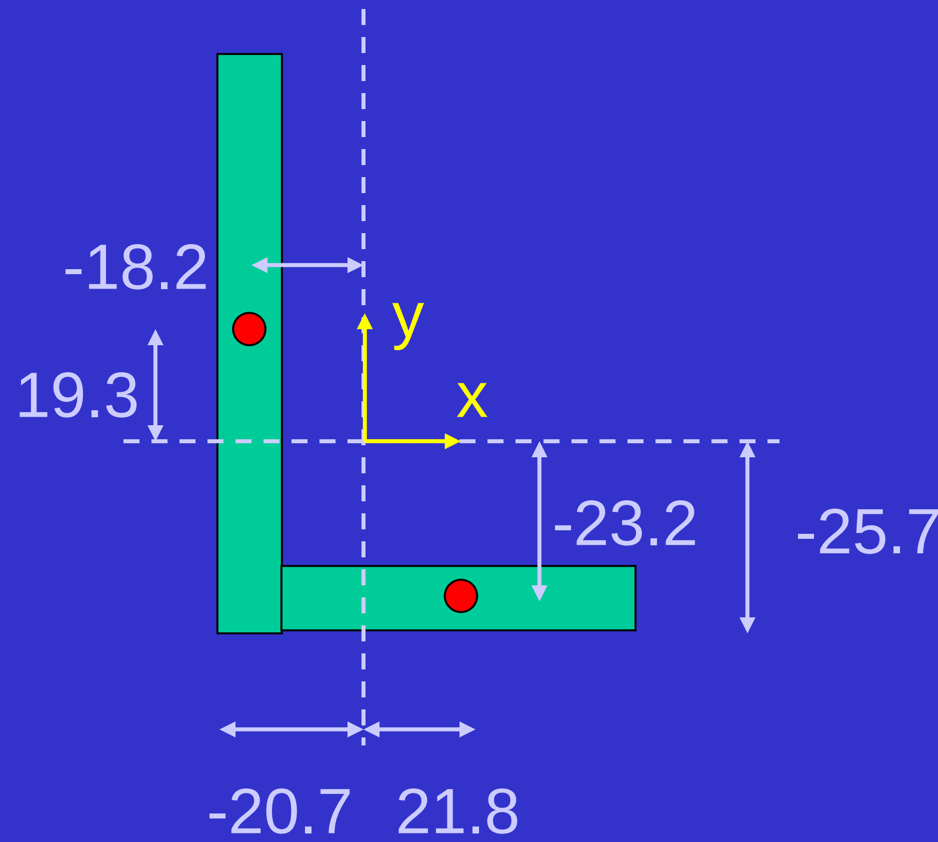
Now find Ixx, Iyy and Ixy about the centroidal axes



$$I_{xx} = \frac{5 \times 90^3}{12} + \left(5 \times 90 \times 19.3^2\right) + \frac{75 \times 5^3}{12} + \left(5 \times 75 \times -23.2^2\right)$$

$$I_{xx} = 673991\, mm^4 = 6.74 \times 10^5\, mm^4$$

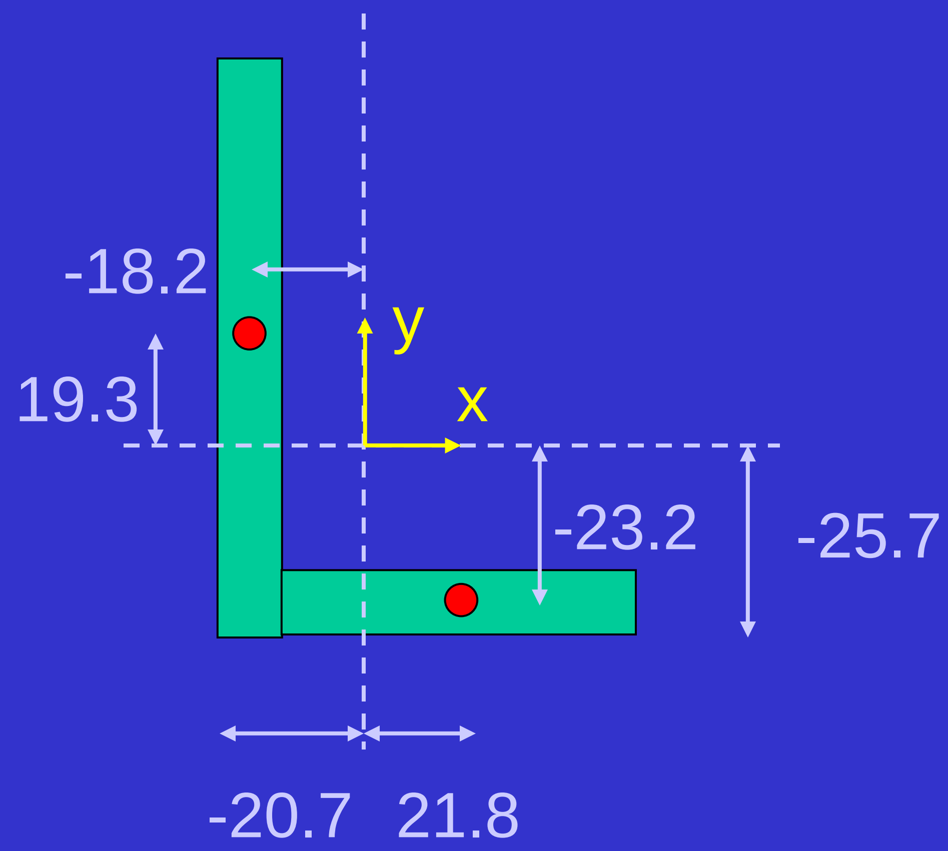
Biaxial bending of asymmetric section



$$I_{yy} = \frac{90 \times 5^3}{12} + \left(5 \times 90 \times -18.2^2\right) + \frac{5 \times 75^3}{12} + \left(5 \times 75 \times 21.8^2\right)$$

$$I_{yy} = 503991\ mm^4 = 5.04 \times 10^5\ mm^4$$

Biaxial bending of asymmetric section



$$I_{xy} = \left(5 \times 90 \times -18.2 \times 19.3\right) + \left(5 \times 75 \times -23.2 \times 21.8\right)$$

$$I_{xy} = -347727\ mm^4 = -3.48 \times 10^5\ mm^4$$

Biaxial bending of asymmetric section

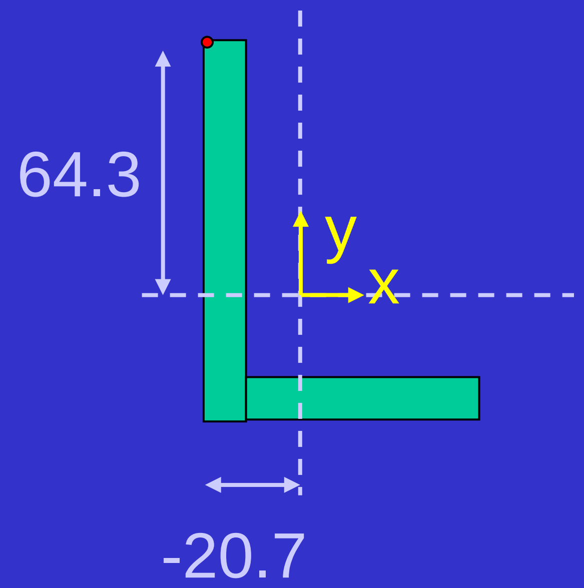
$$\sigma_z = \frac{\left(M_x I_{yy} - M_y I_{xy}\right)}{\left(I_{yy} I_{xx} - I_{xy}^{\;2}\right)} y + \frac{\left(-M_x I_{xy} + M_y I_{xx}\right)}{\left(I_{yy} I_{xx} - I_{xy}^{\;2}\right)} x$$

Hogging +ve, tension +ve

M_x = 5.0 kN.m, M_y = 0

Biaxial bending of asymmetric section

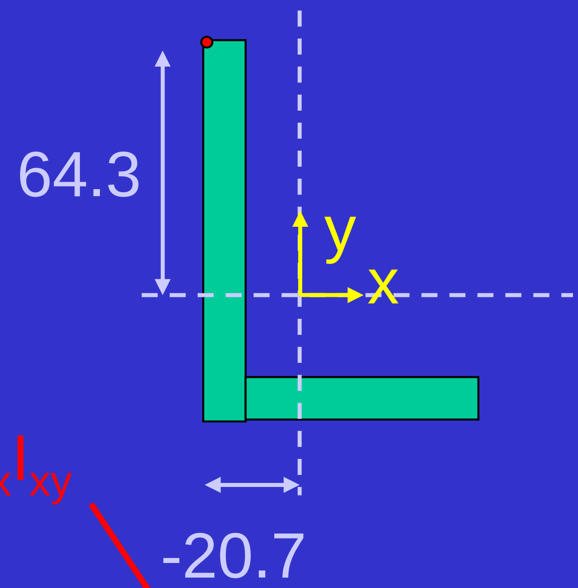
Top-left corner



$$\sigma_z = \frac{\left(5\times10^6 \times 5.04\times10^5\right)}{\left(5.04\times6.74-3.48^2\right)\times10^{10}}\times64.3 + \frac{\left(-5\times10^6 \times \left[-3.45\times10^5\right]\right)}{\left(5.04\times6.74-3.48^2\right)\times10^{10}}\times\left[-20.7\right]$$

Biaxial bending of asymmetric section

Top-left corner



$M_x I_{yy}$

$M_x I_{xy}$

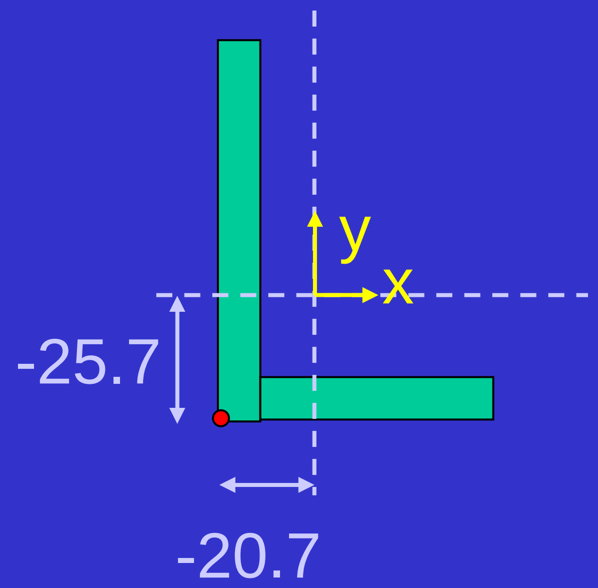
$$\sigma_z = \frac{2.52\times10^{12}}{2.20\times10^{11}}\times64.3+\frac{1.73\times10^{12}}{2.20\times10^{11}}\times[-20.7]$$

$I_{xx}I_{yy}-I_{xx}^2$

$$\sigma_z = 738-163 \; = 575 \; N/mm^2$$

Biaxial bending of asymmetric section

Bottom-left corner

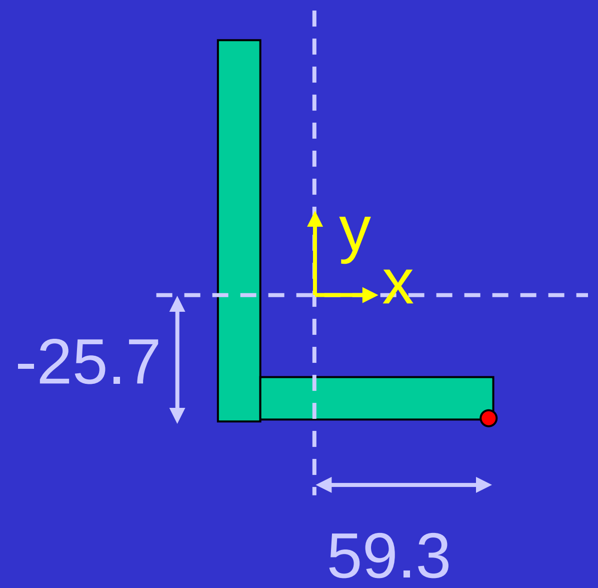


$$\sigma_z = \frac{2.52\times10^{12}}{2.20\times10^{11}}\times[-25.7] + \frac{1.73\times10^{12}}{2.20\times10^{11}}\times[-20.7]$$

$$\sigma_z = -295 - 163 \;\; = -458 \; N/mm^2$$

Biaxial bending of asymmetric section

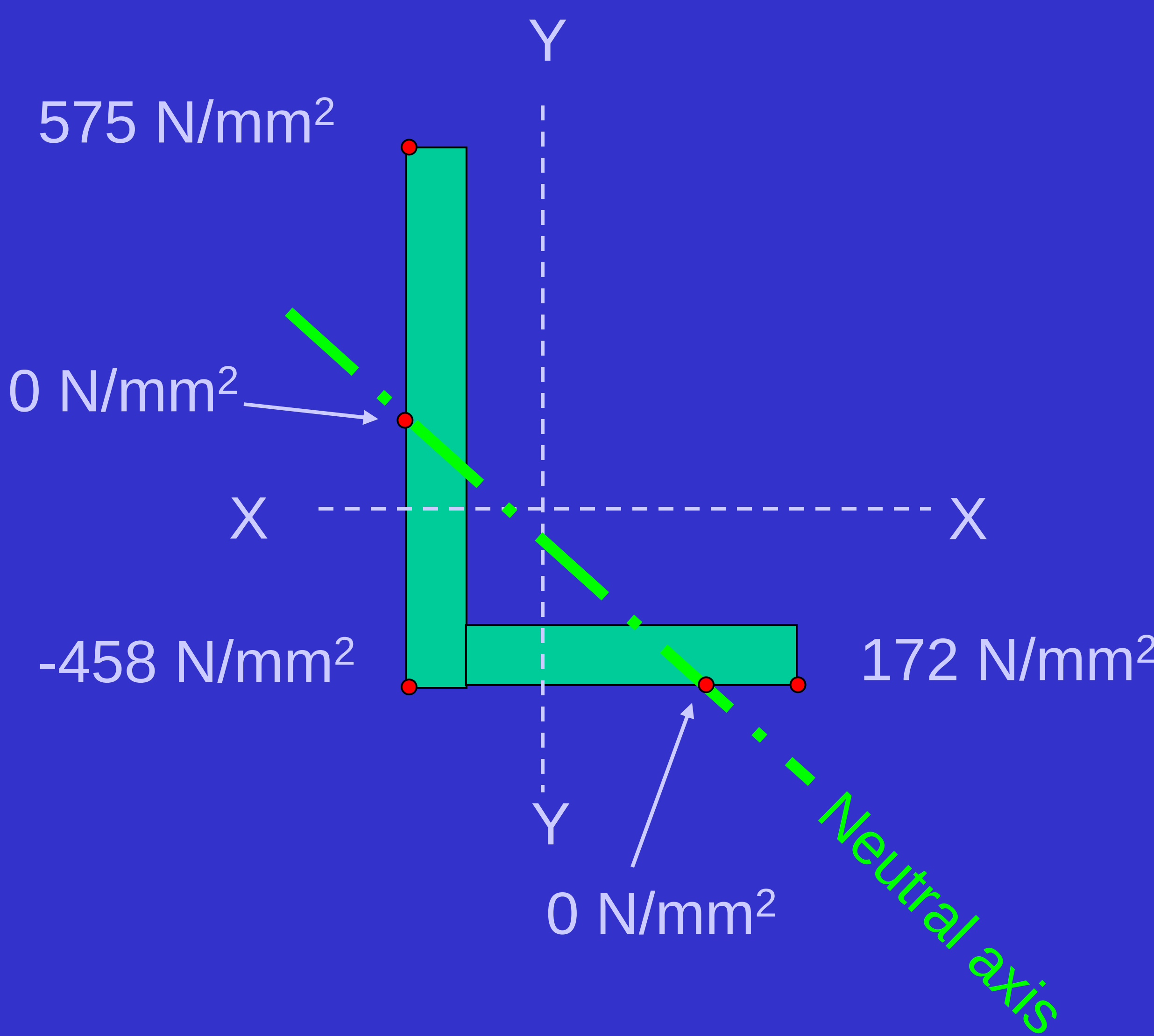
Bottom-right corner



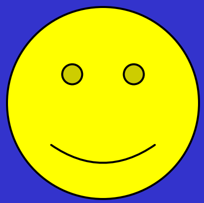
$$\sigma_z = \frac{2.52\times10^{12}}{2.20\times10^{11}}\times[-25.7]+\frac{1.73\times10^{12}}{2.20\times10^{11}}\times59.3$$

$$\sigma_z = -295+467 \quad =172\ N/mm^2$$

Biaxial bending of asymmetric section



Biaxial bending of asymmetric section



Alternative method …

Biaxial bending of asymmetric section

It can be shown that we can use the symmetric equation

$$\sigma_z = \frac{M_x}{I_{xx}} y + \frac{M_y}{I_{yy}} x$$

for asymmetric sections, if we replace M_x and M_y by $\overline{M}_x$ and $\overline{M}_y$, where :

$$\overline{M}_x = \frac{M_x - M_y \times I_{xy} / I_{xx}}{1 - I_{xy}^{\;2} / \left(I_{xx} I_{yy}\right)}$$

$$\overline{M}_y = \frac{M_y - M_x \times I_{xy} / I_{yy}}{1 - I_{xy}^{\;2} / \left(I_{xx} I_{yy}\right)}$$

Biaxial bending of asymmetric section

In the example: M_x = 5.0 kN.m, M_y = 0

$$I_{xx} = 6.74 \times 10^5\, mm^4 \qquad I_{yy} = 5.04 \times 10^5\, mm^4$$

$$I_{xy} = -3.48 \times 10^5\, mm^4$$

$$\overline{M}_x = \frac{5.0}{1 - [-3.48]^2 / (6.74 \times 5.04)}$$ = 7.77 kN.m

$$\overline{M}_y = \frac{-5.0 \times [-3.48/6.74]}{1 - [-3.48]^2 / (6.74 \times 5.04)}$$ = 4.01 kN.m

Note: even though the applied M_y = 0, there is an effective bending M_y for an asymmetric section !

Biaxial bending of asymmetric section

$$\sigma_z = \frac{\overline{M}_x}{I_{xx}} y + \frac{\overline{M}_y}{I_{yy}} x$$

For top left corner

$$\sigma_z = \frac{7.77 \times 10^6}{6.74 \times 10^5} \times 64.3 + \frac{4.01 \times 10^6}{5.05 \times 10^5} \times [-20.7]$$

$$\sigma_z = 741 - 164$$

Same solution as before